SYSTEMS OF PARTICLES AND MOMENTUM

1. Systems of particles and center of mass
2. Another view of systems
3. Impulse
4. Change in momentum
5. Relating change in momentum to impulse

SYSTEMS OF PARTICLES AND CM

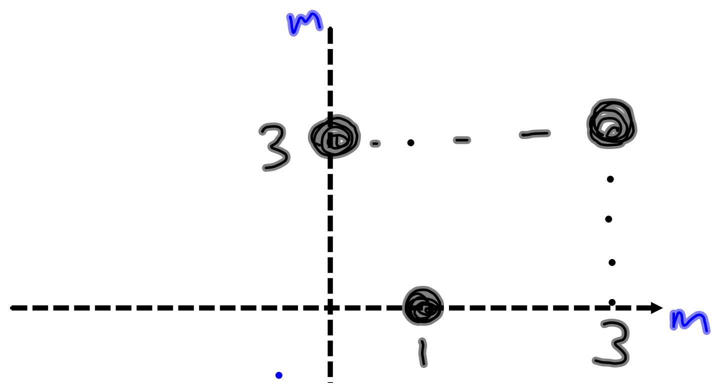
Center of mass = the average position of all the pieces of mass in the system

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{M}$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{M}$$

SYSTEMS OF PARTICLES AND CM
(sample problems)

Find the center of mass of the system



each particle $m = 1\,kg$

$$Y_{cm} = \frac{(1kg)(3) + (1kg)(0) + (1kg)(3m)}{3kg}$$

$$= \frac{3+0+3}{3}$$

$$Y_{cm} = 2m$$

$$X_{cm} = \frac{(1kg)(0) + (1kg)(1 \ldots}{3kg}$$

$$= \frac{0+1+3}{3}$$

$$X_{cm} = \frac{4}{3} = 1.33m$$

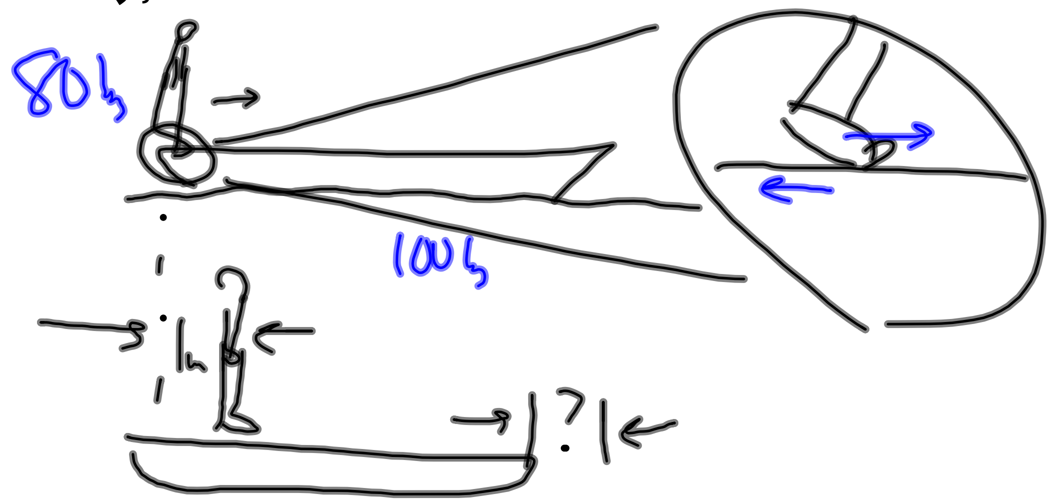
$$(1.33m, 2m)$$

$$\vec{r} = (1.33\hat{i} + 2\hat{j})\,m$$

SYSTEMS OF PARTICLES AND CM

(sample problems)

The person is at the end of the 4 m long boat. Assume the boat's cm is in its middle. If the person moves 1 m to the left (relative to the water), How far over does the boat move?



$$(80\,kg)(1) + (100\,kg)x = 0$$

$$80 + 100x = 0$$

$$100x = -80$$

$$x = -0.8\,m$$

ANOTHER VIEW OF SYSTEMS

Center of mass -> momentum -> Newton's 2nd Law

$$MX_{cm} = m_1x_1 + m_2x_2 + \cdots$$

$$MV_{cm} = m_1v_1 + m_2v_2 + \cdots$$

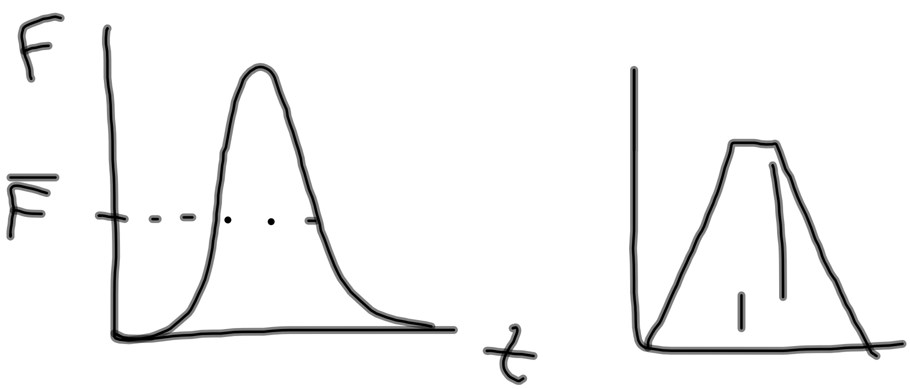
$$Ma_{cm} = m_1a_1 + m_2a_2 + \cdots$$

IMPULSE

$$I = \int F\,dt$$

$= $ area under curve on F vs t

$$I = F_{avg}\,t = \bar{F}\,t$$



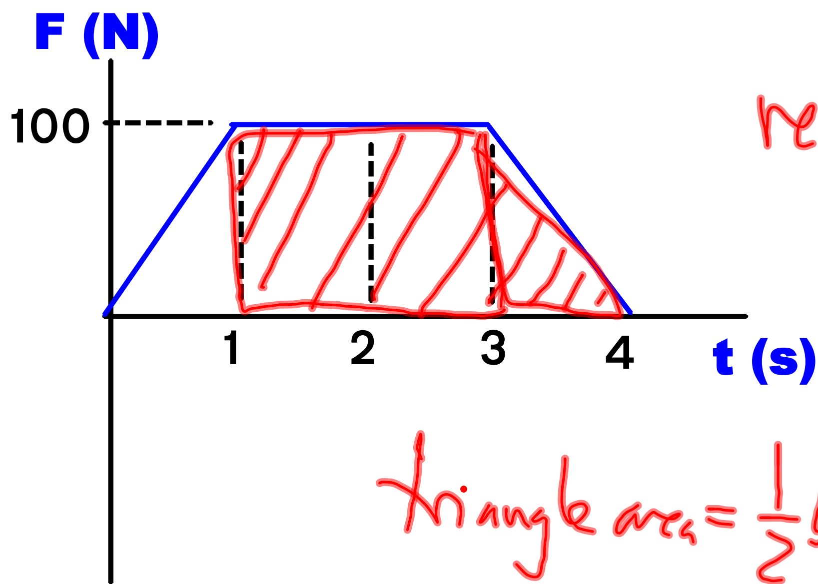
IMPULSE
(sample problem)

Find the impulse delivered by a 200 N force that acts for 0.1 seconds.

$$I = \bar{F}t$$
$$= (200_N)(0.1_s)$$
$$= 20\,Ns$$

IMPULSE
(sample problem)

Find the impulse delivered by the force (graphed below) between t = 1 s and t = 4 s



rectangle $A = bh$

$= (2s)(100N)$

$= 200\ Ns$

triangle area $= \frac{1}{2}bh$

$= \frac{1}{2}(1s)(100N)$

$= 50\ Ns$

$I = 250\ Ns$

IMPULSE
(sample problem)

Find the impulse delivered by the force
F = 4t i - 3 j over from t = 1 s to t = 2 s

$$\int F\,dt = \left[\frac{4t^2}{2} - 3t\right]_{1\,s}^{2\,s}$$

$$\left[2(2)^2\hat{i} - 3(2)\hat{j}\right] - \left[2(1)^2\hat{i} - 3(1)\hat{j}\right]$$

$$8\hat{i} - 2\hat{i} \quad -6\hat{j} + 3\hat{j}$$

$$I = (6\hat{i} - 3\hat{j})\ Ns$$

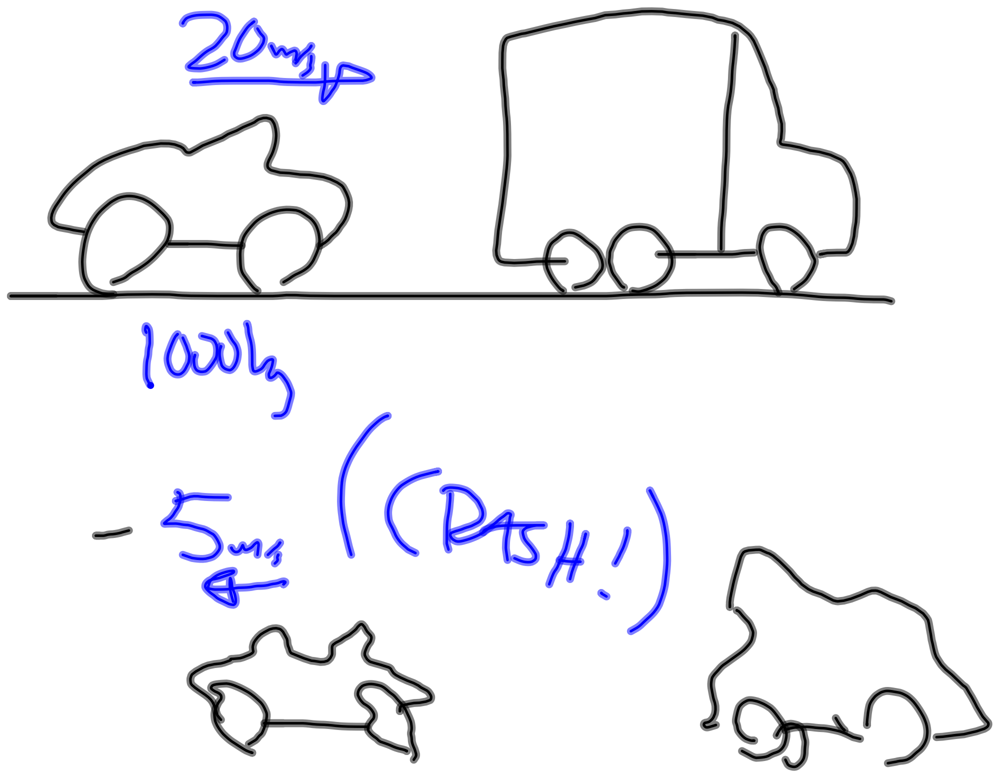
CHANGE IN MOMENTUM

$$\Delta p = p_f - p_i$$

$$\Delta p = mv_f - mv_i$$

CHANGE IN MOMENTUM
(sample problem)

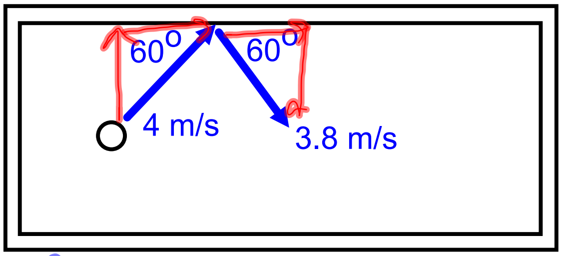
Find the change in momentum of the car.

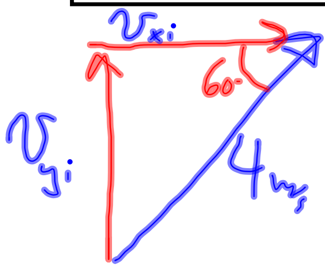


$$\Delta p = mv_f - mv_i = m\Delta v$$
$$= (1000\,kg)(-5\,m/s - 20\,m/s)$$
$$= -25{,}000\,kg\,m/s$$

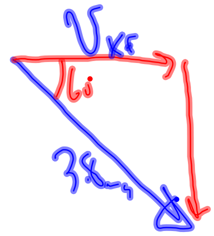
CHANGE IN MOMENTUM (2D)
(sample problem)

Find the change in momentum of the 0.25 billiard ball in ijk notation.





$\Delta p = m \Delta v$



$v_{xi} = 4 \cos 60°$
$= 2 \text{ m/s}$

$v_{yi} = 4 \sin 60°$
$= 3.5 \text{ m/s}$

$v_{xf} = (3.8) \cos 60°$
$= 1.9 \text{ m/s}$

$v_{yf} = 3.8 \sin 60$
$= 3.3 \text{ m/s}$

$\Delta v_x = 1.9 - 2 \text{ m/s} = -0.1 \text{ m/s}$

$\Delta v_y = 3.3 - 3.5 \text{ m/s} = -0.2 \text{ m/s}$

$\Delta v = (-0.1\hat{i} - 0.2\hat{j}) \text{ m/s}$

$\Delta p = m\Delta v = (0.25)(-0.1\hat{i} - 0.2\hat{j}) \text{ m/s}$

$= (-0.025\hat{i} - 0.05\hat{j}) \text{ kg m/s}$

CHANGE IN MOMENTUM = IMPULSE

$$I = \Delta p$$

$$(Ns) = (kg\,m/s)$$

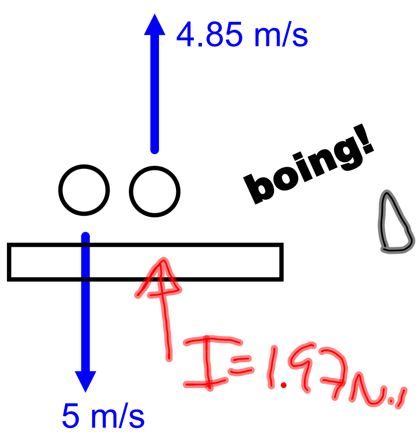
CHANGE IN MOMENTUM = IMPULSE

(sample problem)

The 0.2 kg superball's velocities are shown just before and just after its collision with the surface.

a) Find the impulse delivered to the ball by the floor.

b) If the collision lasts 1/300th of a second, find the average force.



$$\Delta p = m\Delta v$$
$$= m(v_f - v_i)$$
$$= (0.2\,kg)(+4.85\,m/s - -5\,m/s)$$
$$= (0.2\,kg)(9.85\,m/s)$$

b) $I = \Delta p$

$$\bar{F} t = 1.97\,kg\,m/s$$
$$\bar{F}\left(\frac{1}{300}\right) = 1.97\,kg\,m/s$$
$$\bar{F} = 600\,N$$