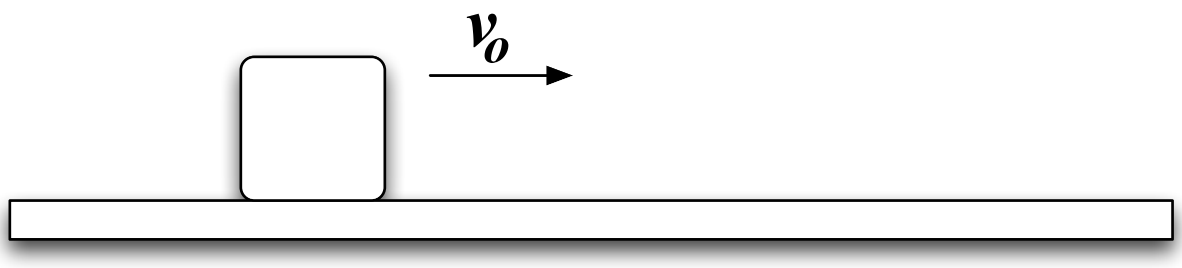


On a cold winter's day an ice cube, mass m, travels with negligible friction over a smooth surface, but experiences a drag force from the air equal to kv opposite to its direction of motion.

a) Sketch a graph of the ice cube's velocity with respect to time.

b) Set up, but do not solve the differential equation for the ice cube's motion.

c) Sketch a graph of the ice cube's position with respect to time.

d) Solve the differential equation to come up with an equation for velocity.

The ice cube is now dropped from rest.

e) Set up, but do not solve the differential equation for the ice cube's motion.

f) Determine the ice cube's terminal velocity.

g) Sketch a graph of the ice cube's velocity with respect to time.

Hints Page

a) Drag velocity vs time graphs always have the same shape. Where should it start? Where should it end up?

b) Always start with $\Sigma F = ma$

c) Where should the position start? What will eventually happen to the speed? What does that mean for position?

d) Separate variables and integrate both sides. If short on time, skip this to save time. It may not be worth the points if there are other parts of other problems you can do more quickly.

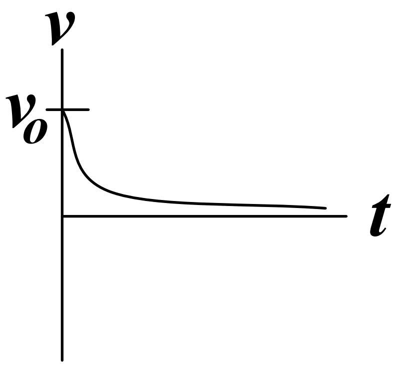
e) Always start with $\Sigma F = ma$

f) What is true of the acceleration when you hit terminal velocity? What does that mean about the forces?

g) Drag velocity vs time graphs always have the same shape. Where should it start? Where should it end up?

Answers Page

a)

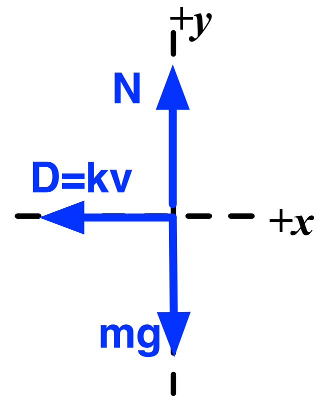


b)

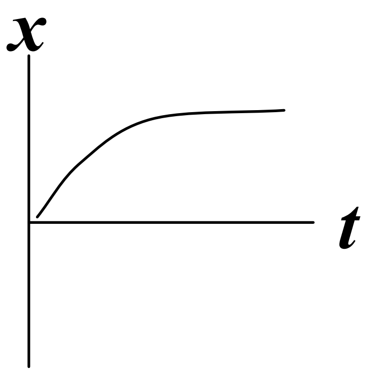
$$\Sigma F = ma$$

$$-kv = ma$$

$$-kv = m\frac{dv}{dt}$$



c)



d)

$$-kv = m\frac{dv}{dt}$$

$$-kvdt = mdv$$

$$\frac{-k}{m}dt = \frac{dv}{v}$$

$$\frac{-k}{m}\int_0^t dt = \int_{v_0}^{v}\frac{dv}{v}$$

$$\frac{-k}{m}[t]_0^t = [\ln(v)]_{v_0}^{v}$$

$$\frac{-k}{m}[t-0] = [\ln(v) - \ln(v_0)]$$

$$\frac{-k}{m}t = \ln\left(\frac{v}{v_0}\right)$$

$$e^{\frac{-k}{m}t} = \frac{v}{v_0}$$

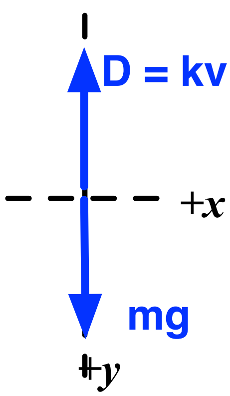
$$v = v_0 e^{\frac{-k}{m}t}$$

e)

$$\Sigma F = ma$$

$$mg - kv = ma$$

$$mg - kv = m\frac{dv}{dt}$$



f)

$$mg - kv = 0$$

$$kv = mg$$

$$v = \frac{mg}{k}$$

g)