UNIFORM CIRCULAR MOTION

(constant speed over any part of a circular path)

1. **Tracking objects around a circle.**

2. **Tangential velocity**

3. **Centripetal acceleration**

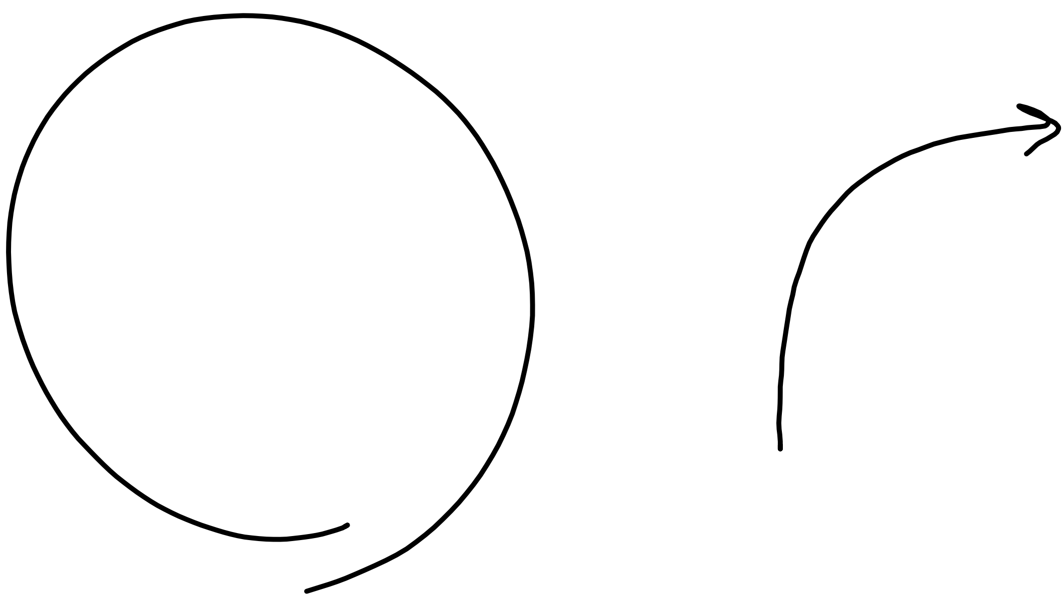
4. **Centripetal force - the net Force**

5. **Centrifugal "force"**

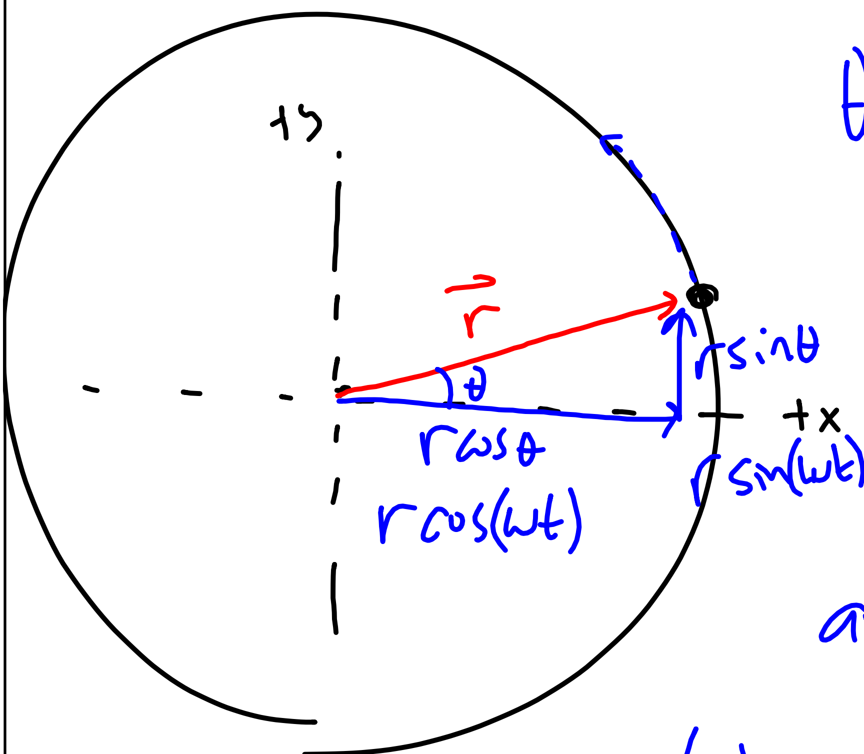
6. **Sample problems**

Uniform Circular Motion

Motion at a constant speed around a circle or an arc of a circle.



Tracking an Object in a Circle



θ in radians

$$\theta = \left(\frac{rad}{s}\right)(s)$$

$$\theta = \omega t$$

angular v

ω = angular velocity

$$\vec{r} = r\cos(\omega t)\hat{i} + r\sin(\omega t)\hat{j}$$

More about ω

"omega" not "double-u"

$$\omega = \frac{d\theta}{dt}$$

$$\omega_{avg} = \frac{2\pi}{T}$$

time for one revolution

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

Units on ω

ω in $\frac{rad}{s}$

ω in rpms

$\frac{rev}{min}$

20 rpms $20 \frac{\cancel{rev}}{\cancel{min}} \times \frac{1\ \cancel{min}}{60\ sec} \times \frac{2\pi\ rad}{1\ \cancel{rev}} = \frac{2}{3}\pi\ rad/s$

$$\vec{r} = 2\cos(3t)\hat{i} + 2\sin(3t)\hat{j}$$

r ω

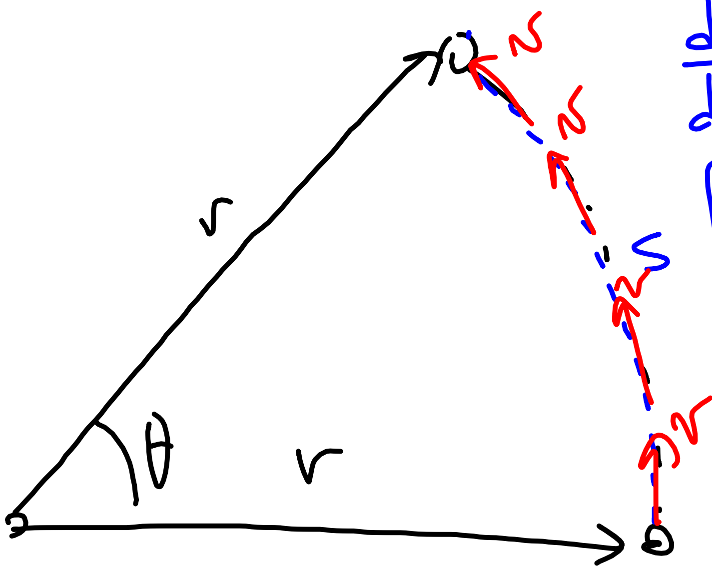
$r = 2\,m$

$\omega = 3\,\frac{rad}{s}$

Distance around the circle, angle, and radius

$$circumference = 2\pi r$$

$$s = r\theta$$



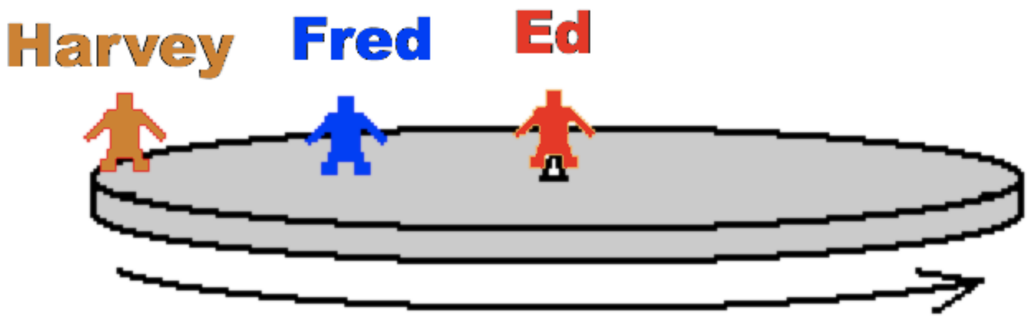
$$\frac{d}{dt}\left(s = r\theta\right)$$

$$\boxed{v = r\omega}$$

linear velocity

ang velocity

$$\left(\frac{m}{s}\right) = (m)\left(\frac{rad}{s}\right)$$



On a rotating object, all positions have the same ω

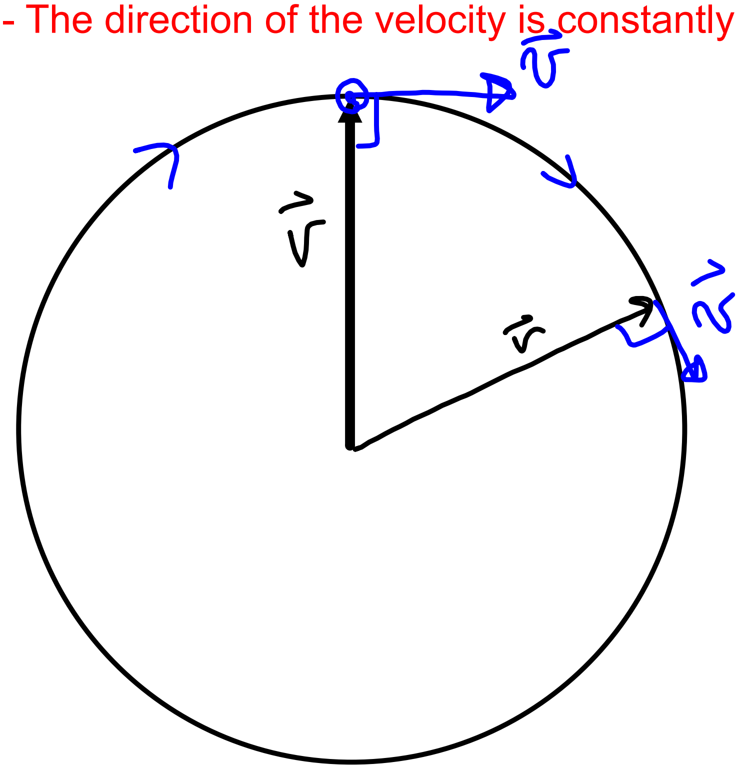
but $v = r\omega$

Further out is faster linear v

$$v = r\left(\frac{2\pi}{T}\right) = \frac{2\pi r}{T}$$

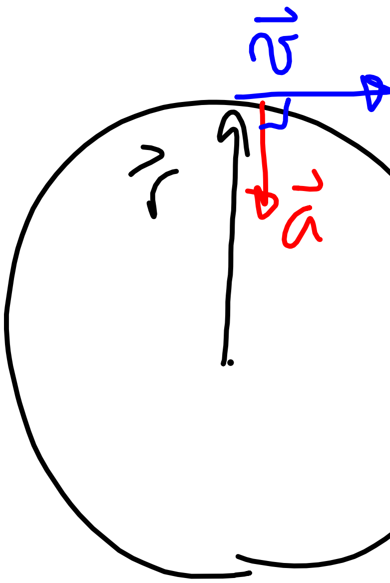
1. Tangential velocity

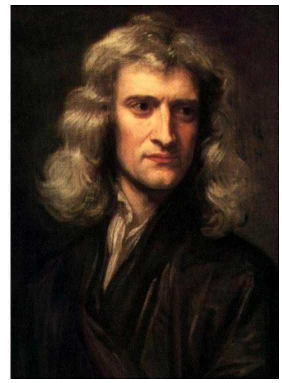
- Velocity is tangent to the circle
- The magnitude of the velocity is constant
- The direction of the velocity is constantly changing



2. Centripetal acceleration

(always toward the center)





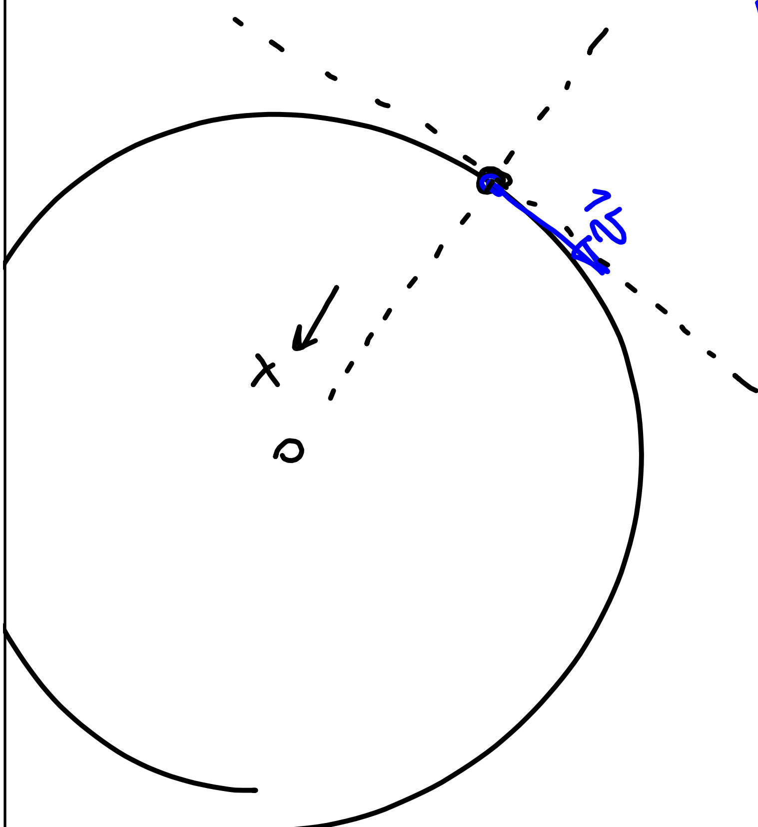
$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r}$$

$$a_c = r\omega^2$$

Newton figured out that the triangle formed by the radius & velocity is similar to the triangle formed by velocity and acceleration.

Centripetal Force

→ net force keeping object in unif. cir. mot.



3. Centripetal force - the net Force

$$\sum F_c = m a_c$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

force toward the center of the circle (along radius)

$$\sum F_\perp = 0$$

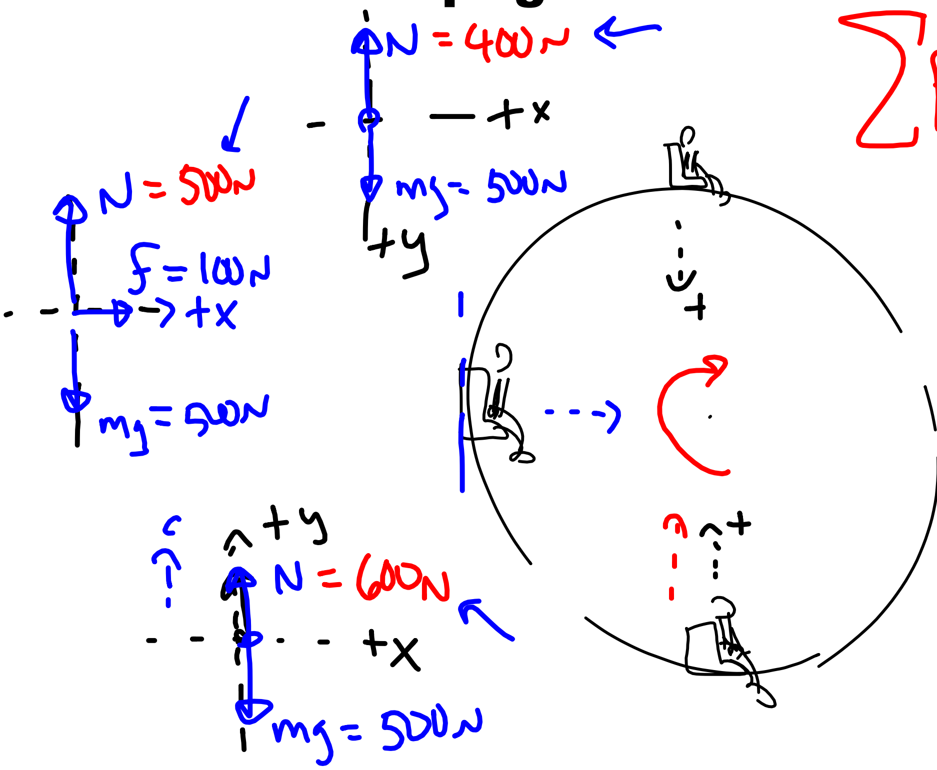
forces tangent to circle (perpendicular to radius)

$\frac{mv^2}{r}$ or $mr\omega^2$ is sometimes called the centripetal force - it is the net force required for uniform circular motion. It must point toward the center of the circle

upright circles

$$mg = 500N$$

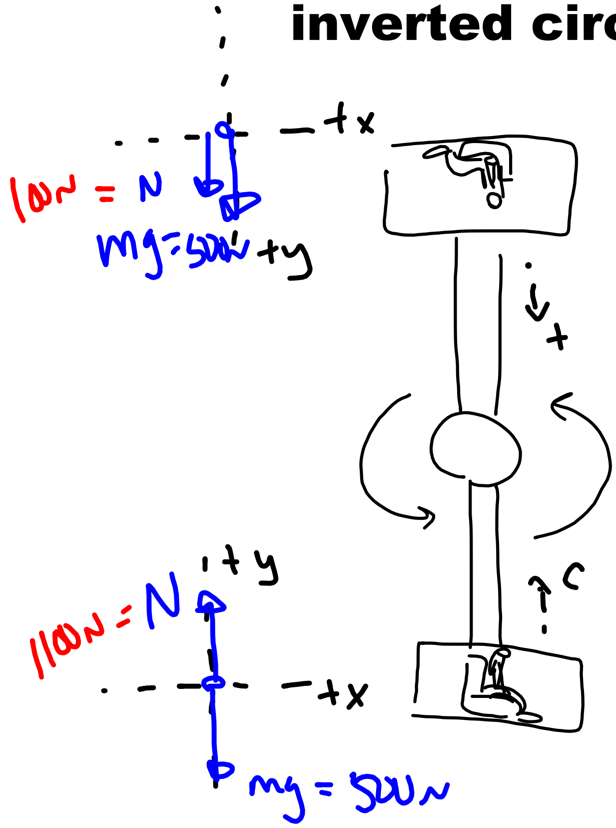
$$\sum F = \frac{mv^2}{r} = 100N$$



inverted circles

$mg = 500N$

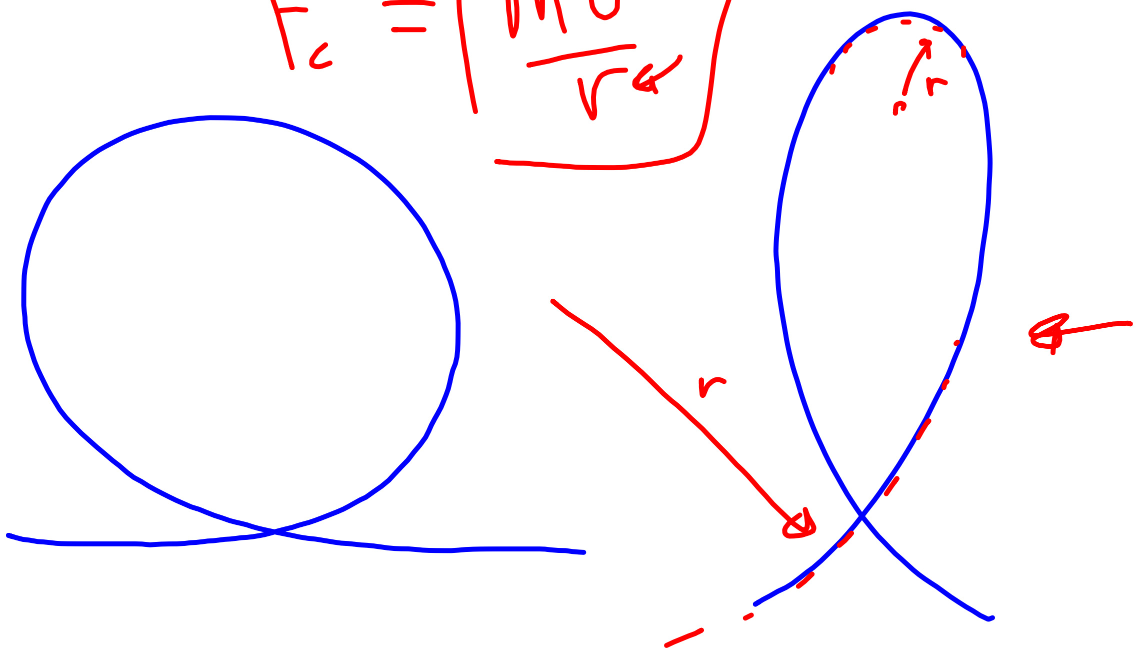
$\sum F = \frac{mv^2}{r} = 600N$



$N - mg = ma_c$

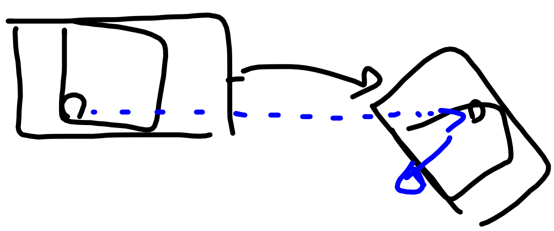
$N - 500 = 600$

$$F_c = \boxed{\frac{mv^2}{r}}$$

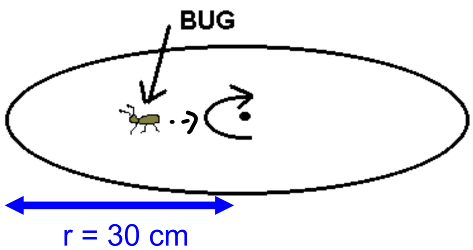


5. Centrifugal "force"

When you are moving in a circle, a force appears to pull objects outward

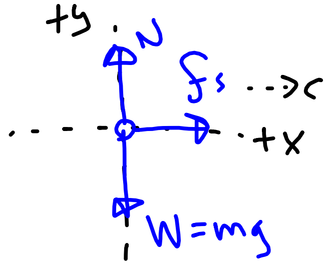


a. unbanked turns



The bug moves toward the outside of a turntable that rotates once every 0.77 seconds. T

The bug has a coefficient of static friction of 0.5 with the turntable. At what point is the bug flung off?



$$f_{s(max)} = \mu_s N$$

$$\sum F_c = ma_c \qquad \sum F_\perp = 0$$

$$f_s = ma_c \qquad N - mg = 0$$

$$\mu_s N = mr\omega^2 \qquad N = mg$$

$$\mu_s \cancel{m}g = \cancel{m}r\omega^2$$

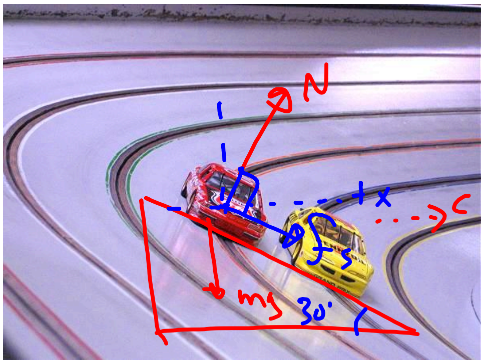
$$\mu_s g = r\omega^2$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.77\,s} = 8.16\,\text{rad/s}$$

$$r = \frac{\mu_s g}{\omega^2} = \frac{(0.5)(10)}{(8.16)^2} = 0.075\,\text{m}$$

7.5 cm

b. banked turns (on ground)

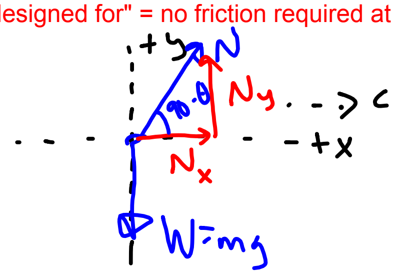


This 30° banked turn is designed for 25 m/s

a. What is the radius of the turn?

b. If the coefficient of static friction between the tires and road is 0.90, how fast can a car take the turn without sliding?

"designed for" = no friction required at that speed



$$\sum F_c = ma_c \qquad \sum F_\perp = 0$$

$$N\cos(90-\theta) = \frac{mv^2}{r} \qquad N\sin(90-\theta) - mg = 0$$

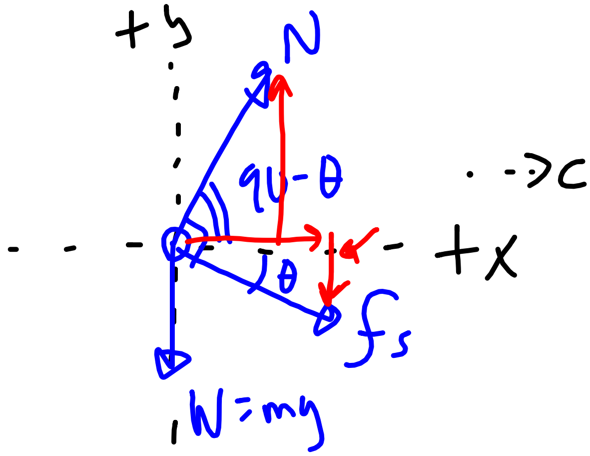
$$N\sin\theta = \frac{mv^2}{r} \qquad N\cos\theta = mg$$

$$N = \frac{mg}{\cos\theta}$$

$$\frac{\cancel{m}g\sin\theta}{\cos\theta} = \frac{\cancel{m}v^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

$$r = \frac{v^2}{g\tan\theta} = \frac{(25)^2}{(10)(0.577)} = 108\,m$$



$$\sum F_c = ma_c \qquad \sum F_\perp = 0$$

$$N\cos(90-\theta) + f_s\cos\theta = ma_c$$

$$N\sin\theta + \mu_s N\cos\theta = \frac{mv^2}{r}$$

$$N(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{r}$$

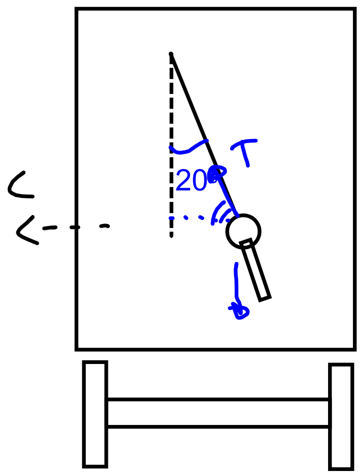
$$N\sin(90-\theta) - f_s\sin\theta - mg = 0$$

$$N\cos\theta - \mu_s N\sin\theta = mg$$

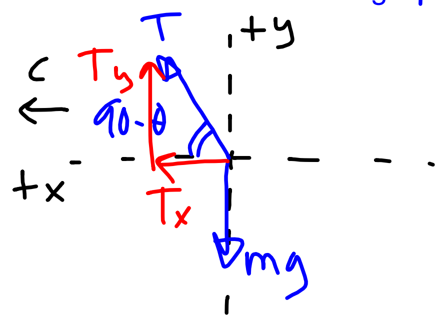
$$N(\cos\theta - \mu_s\sin\theta) = mg$$

$$\boxed{v = 57.6\,\tfrac{m}{s}}$$

c. banked turns (in air)



You are in a train rounding a turn of radius 120 m at a constant speed. You hold your lanyard and keys and note that it makes a 20° angle with the vertical. How fast is the train moving?



$$\sum F_c = ma_c \qquad \sum F_\perp = 0$$

$$T\cos(90-\theta) = \frac{mv^2}{r} \qquad T\sin(90-\theta) - mg = 0$$

$$T\sin\theta = \frac{mv^2}{r} \qquad T\cos\theta = mg$$

$$T = \frac{mg}{\cos\theta}$$

$$\frac{\cancel{m}g\sin\theta}{\cos\theta} = \frac{\cancel{m}v^2}{r}$$

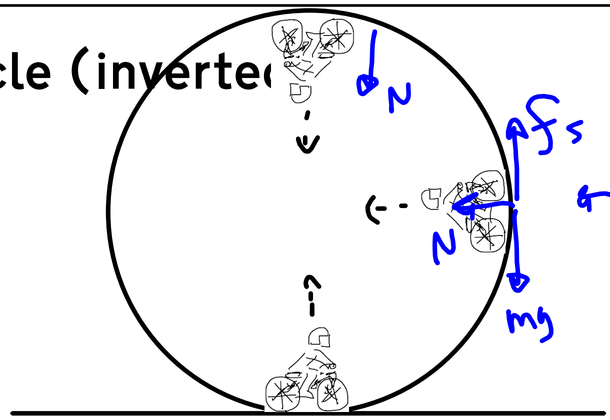
$$g\tan\theta = \frac{v^2}{r}$$

$$v = \sqrt{rg\tan\theta}$$

$$= \sqrt{(120)(10)(\tan 20)}$$

$$\boxed{v = 20.9\,\tfrac{m}{s}}$$

d. vertical circle (inverte



The motorcycle and rider (200 kg) maintain 20 m/s through the loop (radius 10 m.)

Find the Normal force from the track on the motorcycle & rider at the bottom, side & top of the loop.

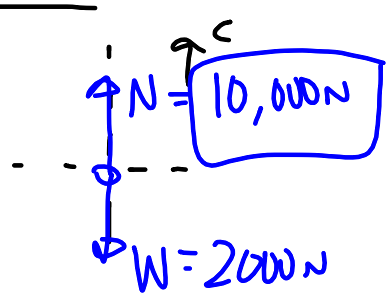
$$\sum F_c = ma_c$$

$$\sum F_c = \frac{mv^2}{r}$$

$$= \frac{(200)(20)^2}{10}$$

$$F_c = 8000\,N$$

bottom

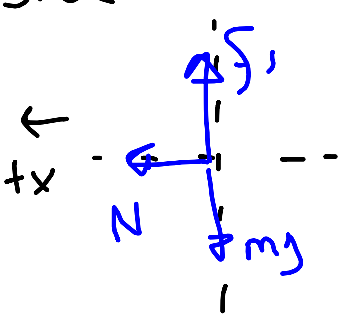


$\boxed{10{,}000\,N}$

$$\sum F_c = 8000$$

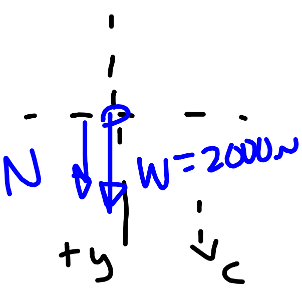
$$N - 2000 = 8000$$

Side



$$\boxed{N = 8000\,N}$$

top



$$N + mg = 8000$$

$$N + 2000 = 8000$$

$$\boxed{N = 6000\,N}$$

e. vertical circle (upright)

At Swatara State Park where my family and I camped, there was a small ferris wheel. I estimate that the radius was about 4 meters, and it went once around every 10 seconds.

a) What is the tangential (linear) velocity of a passenger on the ride?

b) What would the Normal force on a 70 kg passenger be at the top, side and bottom positions?

a) $v = r\omega$

$$= \frac{r(2\pi)}{T} = \frac{4(2\pi)}{10} = 2.5\ \frac{m}{s}$$

b) $\sum F_c = ma_c$

$$\sum F_c = \frac{mv^2}{r} = \frac{(70)(2.5)^2}{4} = 109\ N$$