# UNIFORM CIRCULAR MOTION

(constant speed over any part of a circular path)

1. Tangential velocity

2. Centripetal acceleration

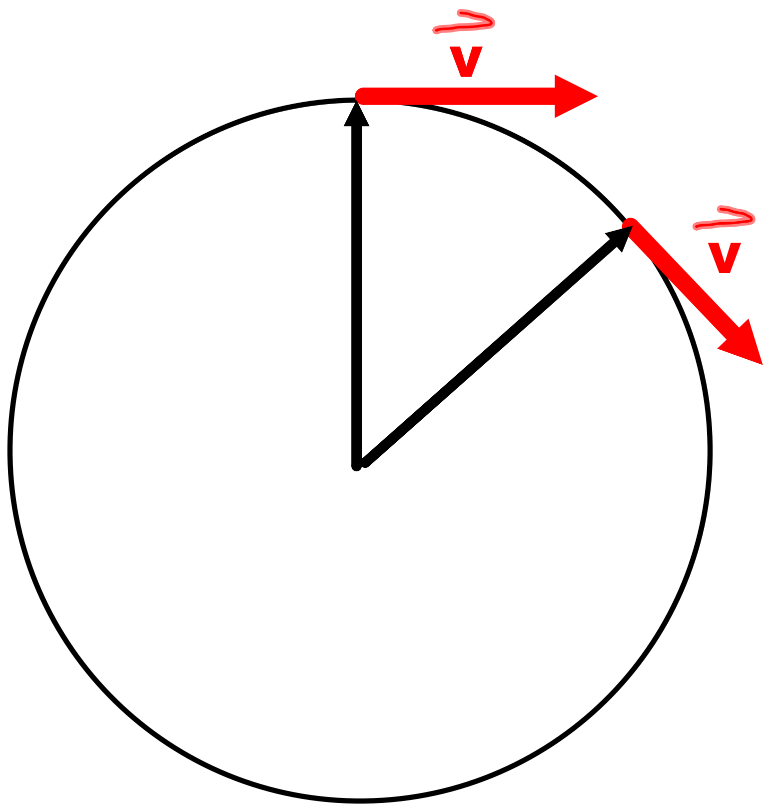
3. Centripetal force - the net Force

4. Sample problems

5. Centrifugal "force"

# 1. Tangential velocity

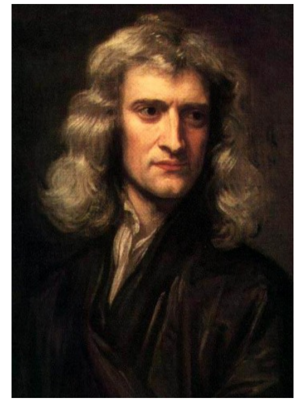
- Velocity is tangent to the circle
- The magnitude of the velocity is constant
- The direction of the velocity is constantly changing

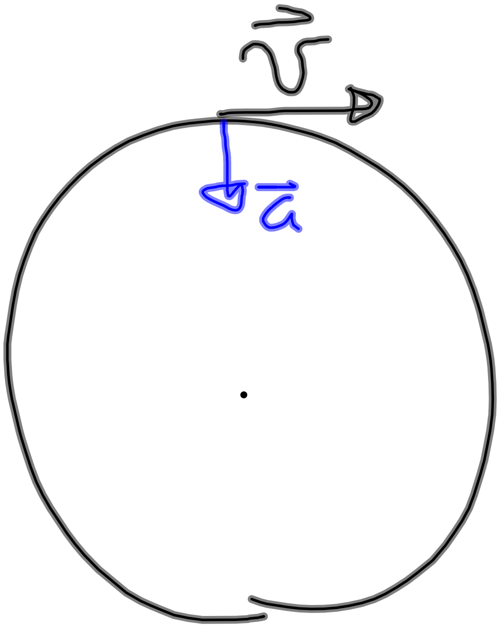



If it goes completely around the circle in time T, then $v = \frac{2\pi r}{T}$

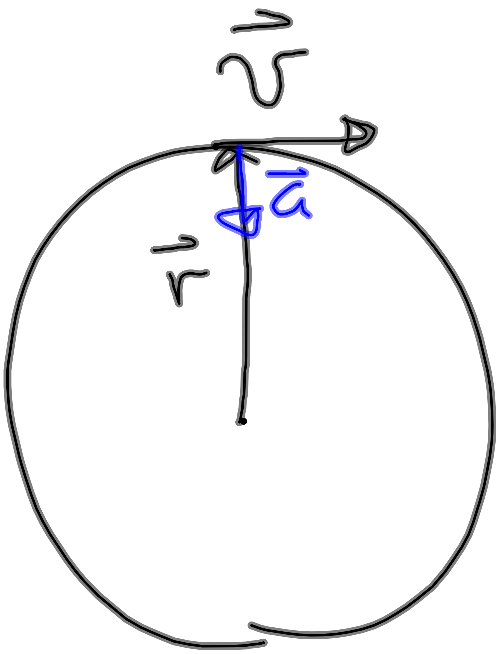
# 2. Centripetal acceleration

(always toward the center)





Newton figured out that the triangle formed by the radius & velocity is similar to the triangle formed by velocity and acceleration.



$$\frac{a}{v} = \frac{v}{r}$$

$$\boxed{a = \frac{v^2}{r}}$$

## 3. Centripetal force - the net Force

$$\sum F_{radial} = m\left(\frac{v^2}{r}\right)$$

force toward the center of the circle (along radius)

$$\sum F_{\perp} = 0$$

forces tangent to circle (perpendicular to radius)

$\frac{mv^2}{r}$ is sometimes called the centripetal force - it is the net force required for uniform circular motion. It must point toward the center of the circle

## 4. Sample problems

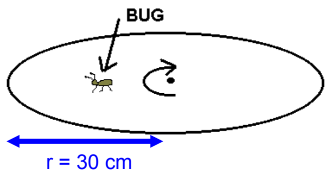
a. unbanked turns

b. banked turns

c. pendulum in vehicle (& airplane)

d. Diavolo (& Ferris Wheel)

## a. unbanked turns




The bug moves toward the outside of a turntable that rotates once every 1.33 seconds.

The bug has a coefficient of static friction of 0.5 with the turntable. At what point is the bug flung off?

$N$ $\qquad \mu_s = 0.5$

$f_s = \mu_s N$ (at slippage)

$mg$

$$\sum F_r = \frac{mv^2}{r} \qquad \sum F_\perp = 0$$

$$\mu_s N = \frac{mv^2}{r} \qquad N - mg = 0$$

$$N = mg$$

$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

Unlike this problem, in most Webassign problems v is given, so you don't have to worry about using $v = \frac{2\pi r}{T}$

$$\mu_s g = \frac{v^2}{r}$$

$$\mu_s g = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

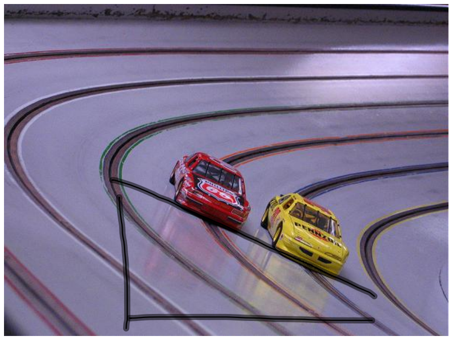
$$\mu_s g = \frac{\frac{4\pi^2 r^{\cancel{2}}}{T^2}}{\cancel{r}}$$

$$\mu_s g = \frac{4\pi^2 r}{T^2}$$

$$\frac{\mu_s g T^2}{4\pi^2} = r = \frac{(0.5)(9.8)(1.33)^2}{4\pi^2}$$

$$\boxed{r = 0.21\,\text{m (fling!)}}$$

# b. banked turns

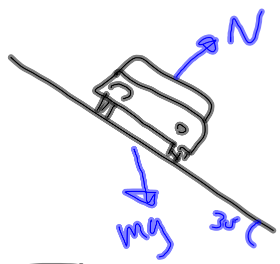


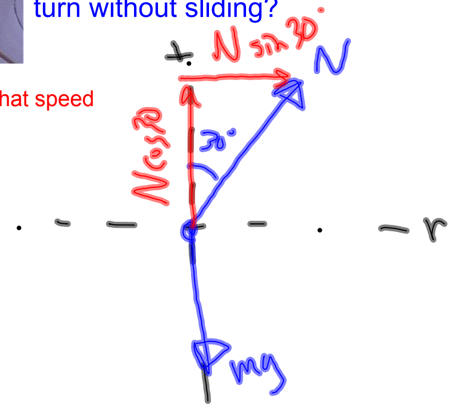
This 30° banked turn is designed for 20 m/s

a. What is the radius of the turn?

b. If the coefficient of static friction between the tires and road is 0.75, how fast can a car take the turn without sliding?

"designed for" = no friction required at that speed




$$\sum F_\perp = 0$$

$$N\cos30 - mg = 0$$

$$N\cos30 = mg$$

$$N = \boxed{\frac{mg}{\cos 30}}$$

$$\sum F_r = \frac{mv^2}{r}$$

$$N\sin30 = \frac{mv^2}{r}$$

$$\frac{mg\sin30}{\cos30} = \frac{mv^2}{r}$$

$$\cancel{m}g\tan30 = \frac{\cancel{m}v^2}{r}$$

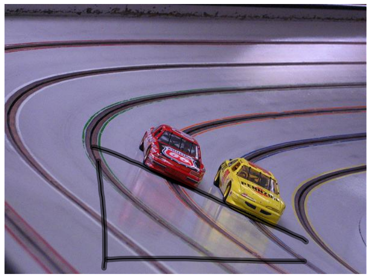
$$g\tan30 = \frac{v^2}{r}$$

$$r = \frac{v^2}{g\tan30}$$

$$= \frac{(20)^2}{9.8\tan30}$$

$$\boxed{r = 70.6\,m}$$

## b. banked turns



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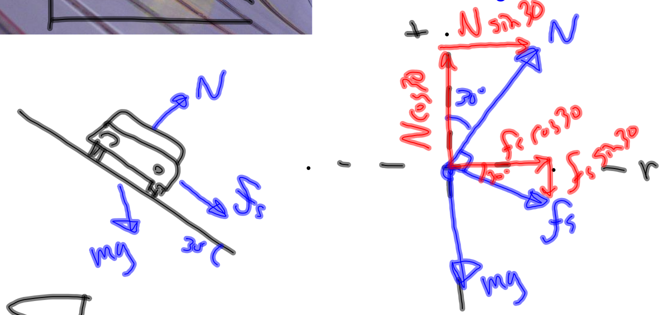



$$\sum F_\perp = 0 \qquad \sum F_r = \frac{mv^2}{r}$$

$$N\cos30 - mg - f_s\sin30 = 0 \qquad N\sin30 + f_s\cos30 = \frac{mv^2}{r}$$

$$N\cos30 - mg - \mu_s N\sin30 = 0 \qquad N\sin30 + \mu_s N\cos30 = \frac{mv^2}{r}$$

$$N\cos30 - \mu_s N\sin30 = mg$$

$$N(\cos30 - \mu_s\sin30) = mg \qquad N(\sin30 + \mu_s\cos30) = \frac{mv^2}{r}$$

$$N = \boxed{\frac{mg}{(\cos30 - \mu_s\sin30)}}$$

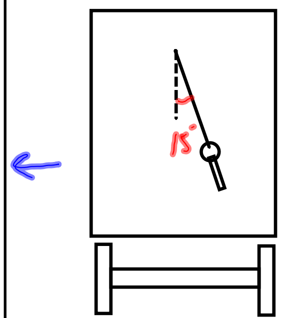
$$\left(\frac{\cancel{m}g}{(\cos30 - \mu_s\sin30)}\right)(\sin30 + \mu_s\cos30) = \frac{\cancel{m}v^2}{r}$$

$$\sqrt{\frac{rg(\sin30 + \mu_s\cos30)}{\cos30 - \mu_s\sin30}} = v$$

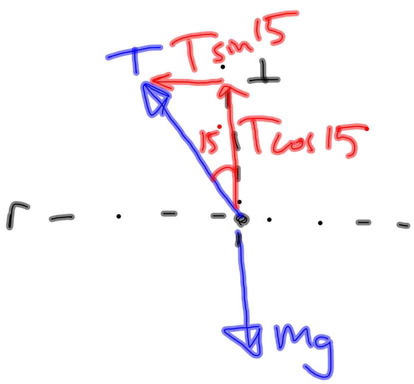
$$\sqrt{\frac{(70.6)(9.8)(0.5 + (0.75)(0.866))}{0.866 - (0.75)(0.5)}} = v$$

$$\boxed{40.2\ m/s = v}$$

## c. pendulum in vehicle (& airplane)




You are in a train rounding a turn of radius 100 m at a constant speed. You hold your lanyard and keys and note that it makes a 15° angle with the vertical. How fast is the train moving?




$$\sum F_{\perp} = 0$$

$$T\cos 15 - mg = 0$$

$$T\cos 15 = mg$$

$$T = \boxed{\frac{mg}{\cos 15}}$$

$$\sum F_r = \frac{mv^2}{r}$$

$$T\sin 15 = \frac{mv^2}{r}$$

$$\frac{mg\sin 15}{\cos 15} = \frac{mv^2}{r}$$

$$\cancel{m}g\tan 15 = \frac{\cancel{m}v^2}{r}$$

$$rg\tan 15 = v^2$$

$$\sqrt{rg\tan 15} = v$$

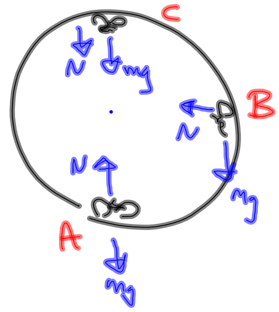
$$\sqrt{(100)(9.8)\tan 15^{\circ}} = v$$

$$\boxed{16.2\ \text{m/s} = v}$$

## d. Diavolo (& Ferris Wheel)


The brave bike rider maintains 12 m/s through the loop (diameter 20 m.)

Find the Normal force from the track on the bike & rider (mass 100 kg) at the bottom, side & top of the loop.




A/

$$\sum F_r = \frac{mv^2}{r}$$

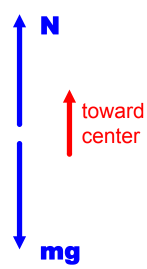
$$N - mg = \frac{mv^2}{r}$$

$$N - (100)(9.8) = \frac{(100)(12)^2}{10}$$

(note that the diameter was 20; therefore the radius is 10)

$N - 980 = 1440$

$\boxed{N = 2420\ \text{N}}$



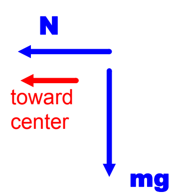

B/

$$\sum F_c = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{r}$$

$$N = \frac{(100)(12)^2}{10}$$

$\boxed{N = 1440\ \text{N}}$




C/

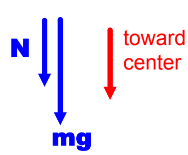
$$\sum F_r = \frac{mv^2}{r}$$

$$N + mg = \frac{mv^2}{r}$$

$$N + (100)(9.8) = \frac{(100)(12)^2}{10}$$

$N + 980 = 1440$

$\boxed{N = 460\ \text{N}}$



## 5. Centrifugal "force"

When you are moving in a circle, a force appears to pull objects outward