Solving the Differential Equation to get v(t)

$$\sum F = ma$$

$$-kv = ma$$

$$-kv = m\frac{dv}{dt}$$

$$-\frac{k}{m}dt = \frac{dv}{v}$$

$$-\frac{k}{m}\int_0^t dt = \int_{v_0}^{v}\frac{dv}{v}$$

$$-\frac{k}{m}\int_0^t dt = \int_{v_0}^{v} v^{-1}\,dv$$

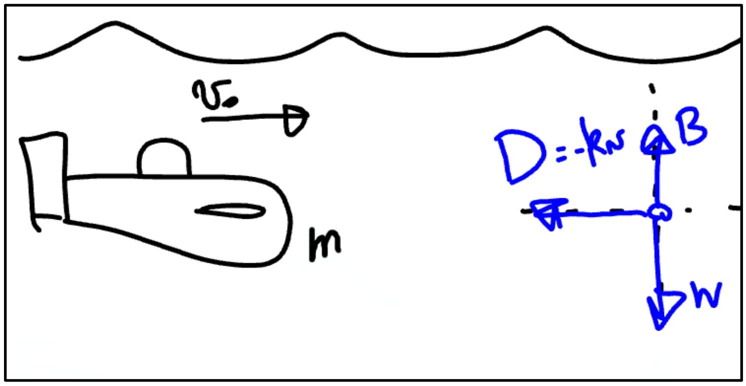
$$-\frac{k}{m}[t]_0^t = [\ln v]_{v_0}^{v}$$

$$-\frac{k}{m}t = \ln v - \ln v_0$$

$$-\frac{k}{m}t = \ln\left(\frac{v}{v_0}\right)$$

$$e^{-\frac{k}{m}t} = \frac{v}{v_0}$$

$$v = v_0 e^{-\frac{k}{m}t}$$



Re-integrating to get x(t)

$$v = v_0 e^{-\frac{k}{m}t}$$

$$\frac{dx}{dt} = v_0 e^{-\frac{k}{m}t}$$

$$dx = v_0 e^{-\frac{k}{m}t}dt$$

$$\int_{x_0}^{x} dx = v_0\int_0^t e^{-\frac{k}{m}t}\,dt$$

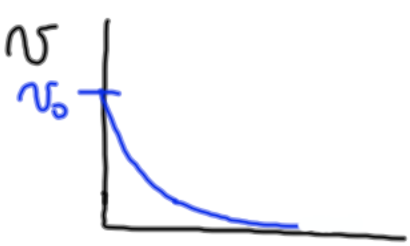
$$[x]_{x_0}^{x} = v_0\left[-\frac{m}{k}e^{-\frac{k}{m}t}\right]_0^t$$

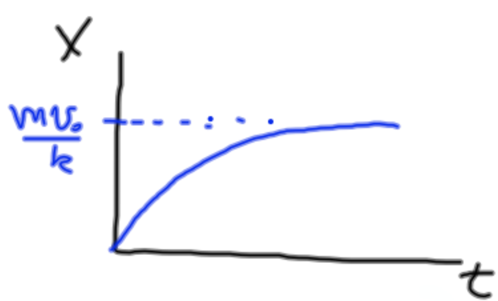
$$x - x_0 = v_0\left[-\frac{m}{k}e^{-\frac{k}{m}t} + \frac{m}{k}e^{-\frac{k}{m}0}\right]$$

$$\Delta x = v_0\left(-\frac{m}{k}e^{-\frac{k}{m}t} + \frac{m}{k}\right)$$

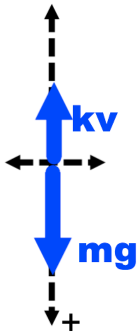
$$\Delta x = \frac{mv_0}{k}\left(-e^{-\frac{k}{m}t} + 1\right)$$

$$\Delta x = \frac{mv_0}{k}\left(1 - e^{-\frac{k}{m}t}\right)$$





Solving the differential equation



The ball (mass m) falls from rest, and experiences a Drag force equal to kv (but in the direction opposite the velocity).

Getting v(t)

$$\sum F = ma$$

$$mg - kv = ma$$

$$mg - kv = m\frac{dv}{dt}$$

$$\frac{1}{m}dt = \frac{dv}{mg - kv}$$

$$\frac{1}{m}\int_0^t dt = \int_0^v \frac{dv}{mg - kv}$$

(u-substitution) let:

$$u = mg - kv$$

$$du = -kdv$$

$$\frac{1}{m}\int_0^t dt = -\frac{1}{k}\int_0^v \frac{du}{u}$$

$$-\frac{k}{m}\int_0^t dt = \int_0^v \frac{du}{u}$$

$$-\frac{k}{m}[t]_0^t = [\ln(u)]_0^v$$

$$-\frac{k}{m}t = [\ln(mg - kv)]_0^v$$

$$-\frac{k}{m}t = [\ln(mg - kv)] - [\ln(mg)]$$

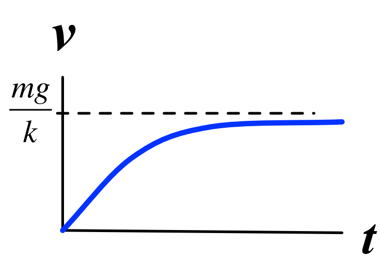
$$-\frac{k}{m}t = \ln\left(\frac{mg - kv}{mg}\right)$$

$$-\frac{k}{m}t = \ln\left(1 - \frac{kv}{mg}\right)$$

$$e^{-\frac{k}{m}t} = 1 - \frac{kv}{mg}$$

$$\frac{kv}{mg} = 1 - e^{-\frac{k}{m}t}$$

$$v = \frac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right)$$



Re-integrating to get x(t)

$$v = \frac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right)$$

$$v = \frac{mg}{k} - \frac{mg}{k}e^{-\frac{k}{m}t}$$

$$\frac{dx}{dt} = \frac{mg}{k} - \frac{mg}{k}e^{-\frac{k}{m}t}$$

$$dx = \frac{mg}{k}dt - \frac{mg}{k}e^{-\frac{k}{m}t}dt$$

$$\int_0^x dx = \frac{mg}{k}\int_0^t dt - \frac{mg}{k}\int_0^t e^{-\frac{k}{m}t}dt$$

$$x = \frac{mg}{k}t - \frac{mg}{k}\left[-\frac{m}{k}e^{-\frac{k}{m}t}\right]_0^t$$

$$x = \frac{mg}{k}t + \frac{m^2 g}{k^2}\left[e^{-\frac{k}{m}t}\right]_0^t$$

$$x = \frac{mg}{k}t + \frac{m^2 g}{k^2}\left[e^{-\frac{k}{m}t} - 1\right]$$