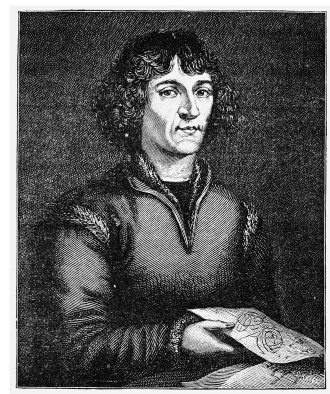
Kepler & Copernicus



Orbits & Kepler's Three Laws of Planetary Motion

1. Planets follow an elliptical path around the sun - the sun is at one focus.

2. A line connecting the planet and the sun sweeps out equal areas in equal times.

3. The square of the orbital period is proportional to the cube of the semi-major axis.

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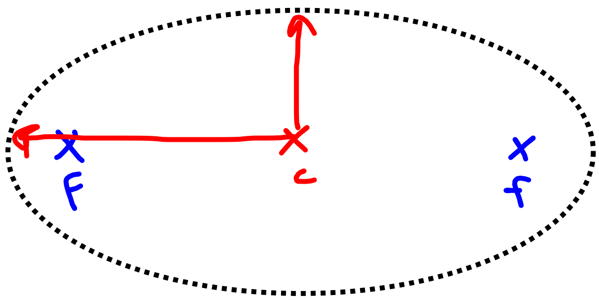
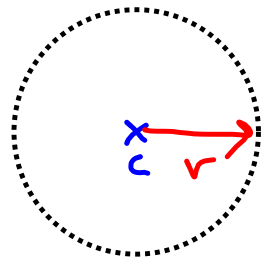
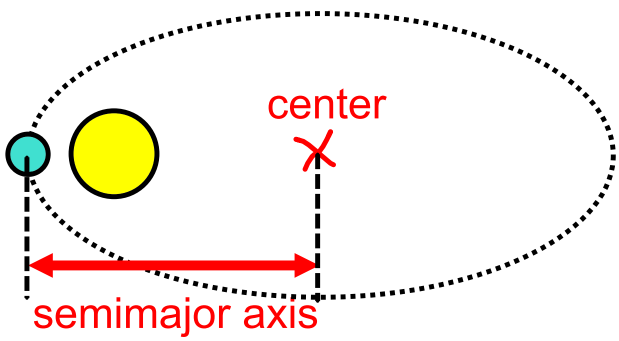
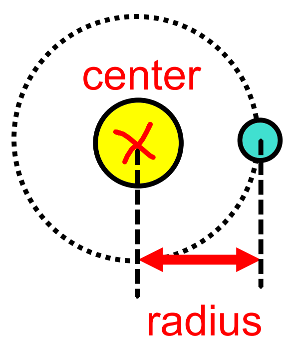
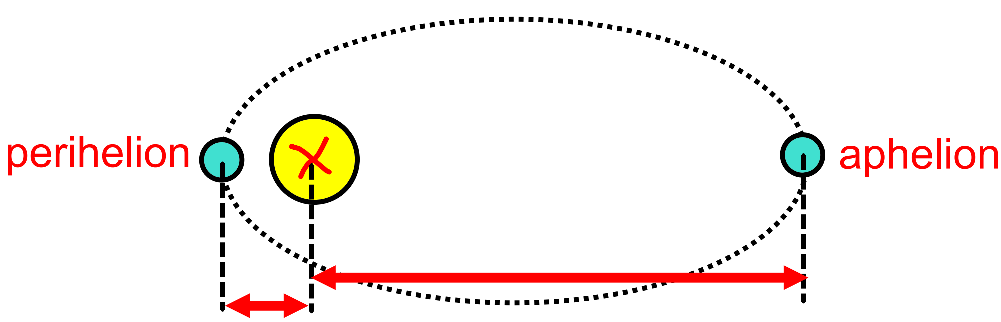


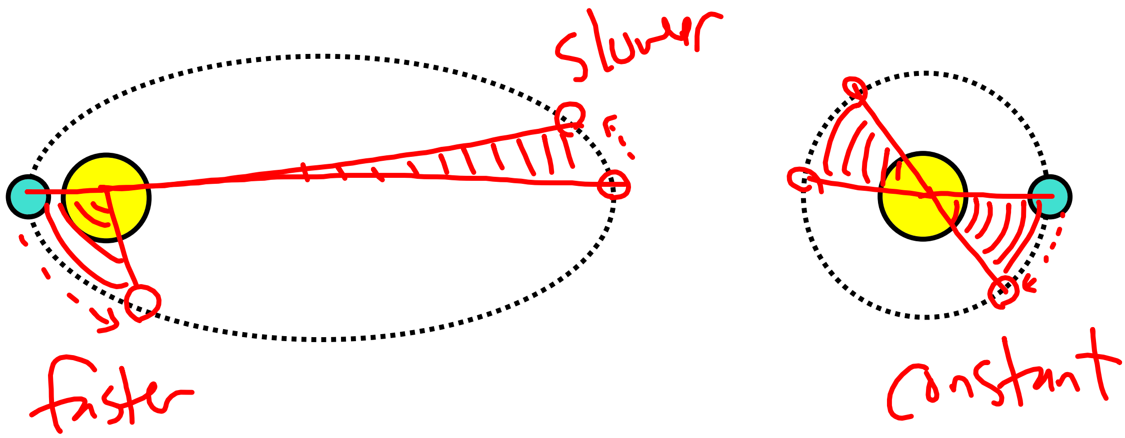




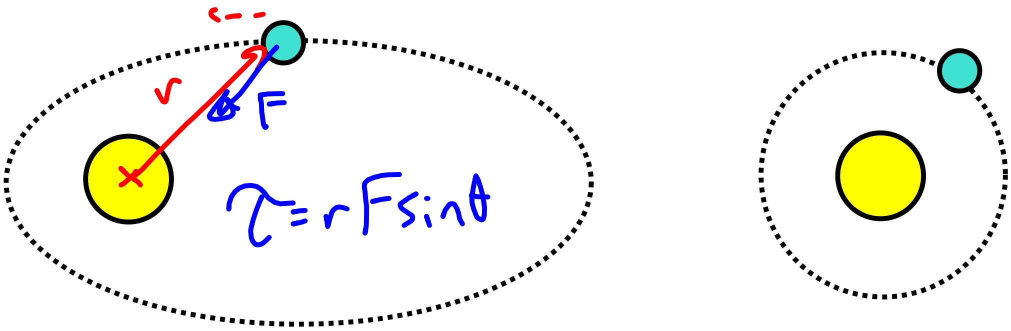


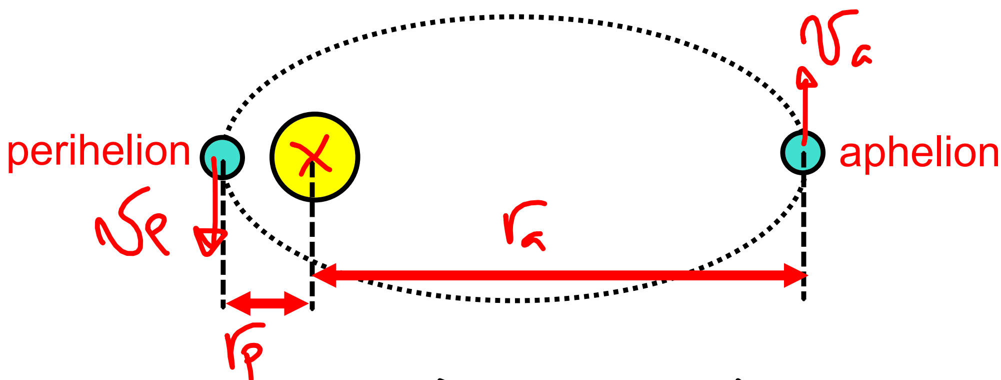


2. A line connecting the planet and the sun sweeps out equal areas in equal times.



Any outside torques? Angular Momentum?





$$\vec{r}_1 \times \vec{p}_1 = \vec{r}_2 \times \vec{p}_2$$

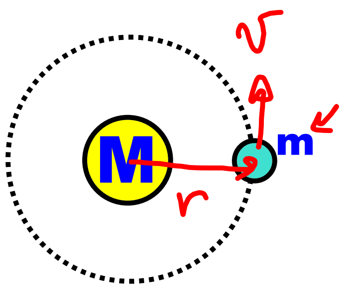
$$r_\perp p = r_\perp p$$

$$r_p \cancel{m} v_p = r_a \cancel{m} v_a$$

$$\boxed{r_p v_p = r_a v_a}$$

3. The square of the orbital period is proportional to the cube of the semi-major axis.

First: Orbital Velocity - Circular Orbit



$$\sum F_c = \frac{mv^2}{r}$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2$$

(Circular)

$$v_{orbital} = \sqrt{\frac{GM}{r}}$$

being orbitted

Re-write the velocity in terms of period

$$v^2 = \frac{GM}{r}$$

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$4\pi^2 r^3 = GMT^2 \text{ (circular)}$$

$$r^3 \propto T^2$$

$$4\pi a^3 = GMT^2 \text{ (elliptical)}$$

semimajor axis

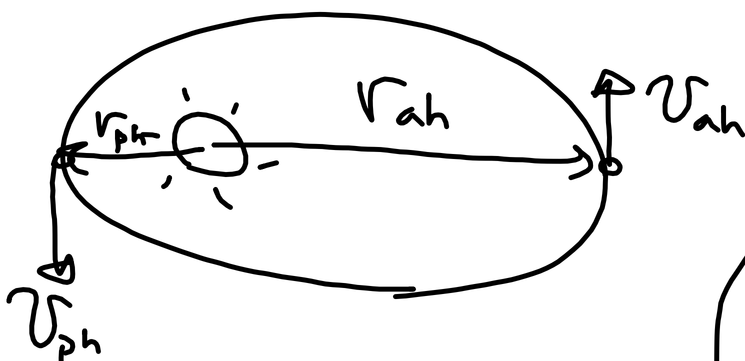
Orbits

$$v = \sqrt{\frac{GM}{r}}$$

$$4\pi^2 r^3 = GMT^2$$

circular orbit

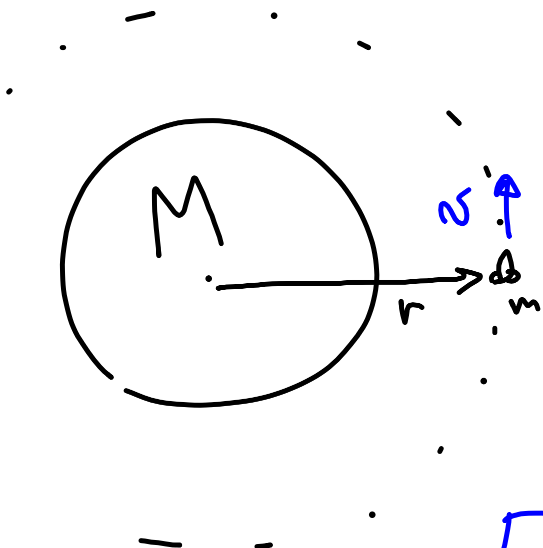
$$4\pi^2 a^3 = GMT^2$$



$$v_{ph} r_{ph} = v_{ah} r_{ah}$$

elliptical

Orbital Energy (circular orbit)



$$E = U + K$$

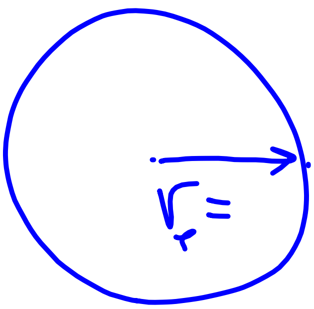
$$E = -\frac{GMm}{r} + \frac{1}{2}mv^2$$

$$E = -\frac{GMm}{r} + \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2$$

$$E = -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r}$$

$$\boxed{E = -\frac{1}{2}\frac{GMm}{r}}$$ circular orbit.

Calculate orbital v and escape v for Earth.
(Assume you start close to Earth's surface.)



$r_e = 6.37\times10^{6}\,m$

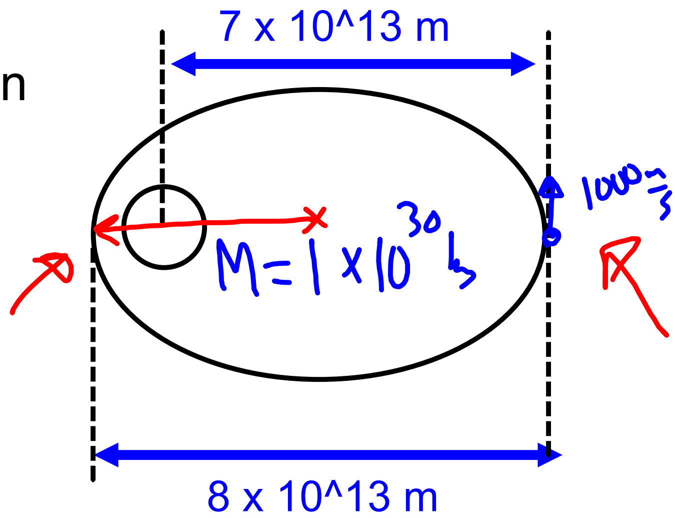
$M_e = 5.98\times10^{24}\,kg$

$$v_{orb} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.37\times10^{6})}} = \boxed{7900\,\frac{m}{s}}$$

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$= \sqrt{2}\,v_{orb} = \sqrt{2}\;7900\,\frac{m}{s} = \boxed{11{,}200\,\frac{m}{s}}$$

An asteroid (orbit shown) has an aphelion speed of 1,000 m/s.
(a) Find the Period of the orbit.
(b) Find the speed at perihelion.



a) $4\pi^2 a^3 = GMT^2$ $\quad a = 4\times10^{13}\,m$

$$4\pi^2(4\times10^{13}) = (6.67\times10^{-11})(1\times10^{30})T^2$$

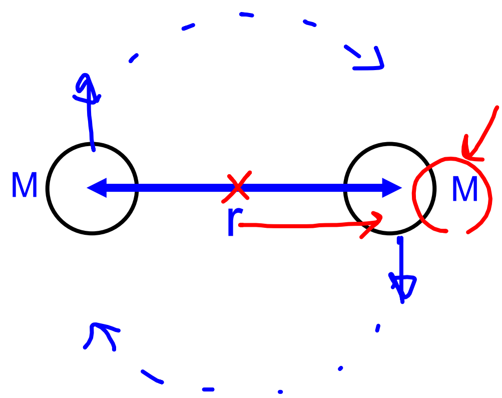
$$\boxed{1.95\times10^{11}\,s = T}$$

b) $r_p v_p = r_a v_a$

$$(1\times10^{13}\,m)v_p = (7\times10^{13}\,m)(1000\,\tfrac{m}{s})$$

$$\boxed{v_p = 7000\,\tfrac{m}{s}}$$

Re-derive the $T^2 \propto r^3$ relationship for a binary star system with identical stars, mass M, with a CM to CM distance of r.



$$\cancel{4\pi^2 r^3 = GMT^2}$$

$$\sum F_c = \frac{Mv^2}{(r/2)}$$

$$\frac{GM\cancel{M}}{r^2} = \frac{\cancel{M}v^2}{(r/2)}$$

$$\frac{GM}{r^{\cancel{2}}} = \frac{2v^2}{\cancel{r}}$$

$$v^2 = \frac{GM}{2r} \rightarrow v_{orb} = \sqrt{\frac{GM}{2r}}$$

$$\left(\frac{\cancel{2}\pi(r/\cancel{2})}{T}\right)^2 = \frac{GM}{2r}$$

$$\frac{\pi^2 r^2}{T^2} = \frac{GM}{2r}$$

$$\boxed{2\pi^2 r^3 = GMT^2}$$

binary system identical stars