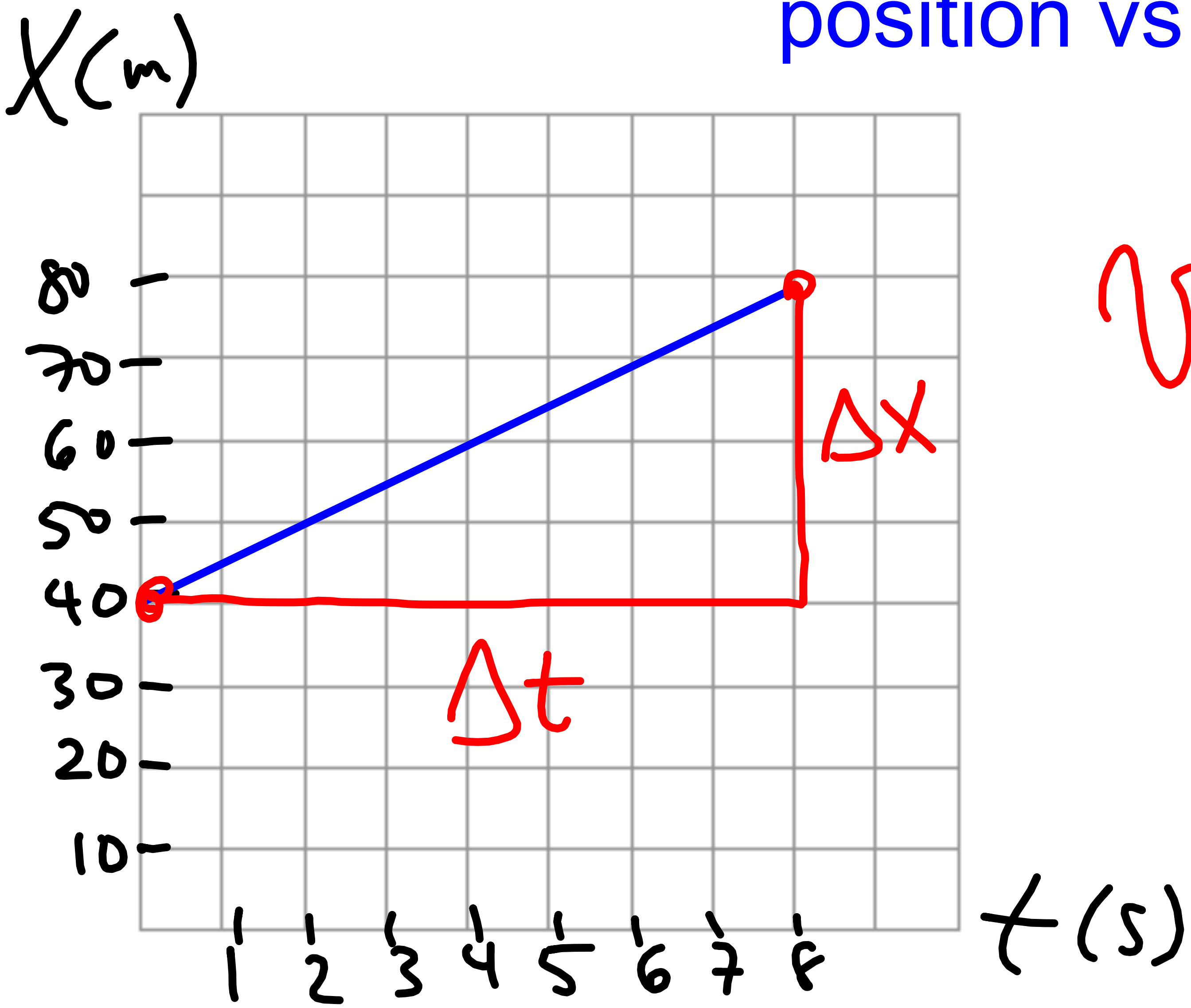
Calculus in Physics

- Slopes: why they are important.
- Finding constant slopes.
- Finding non-constant slopes graphically.
- Finding non-constant slopes with calculus.
- Using calculus in physics.

Why Slope is Important

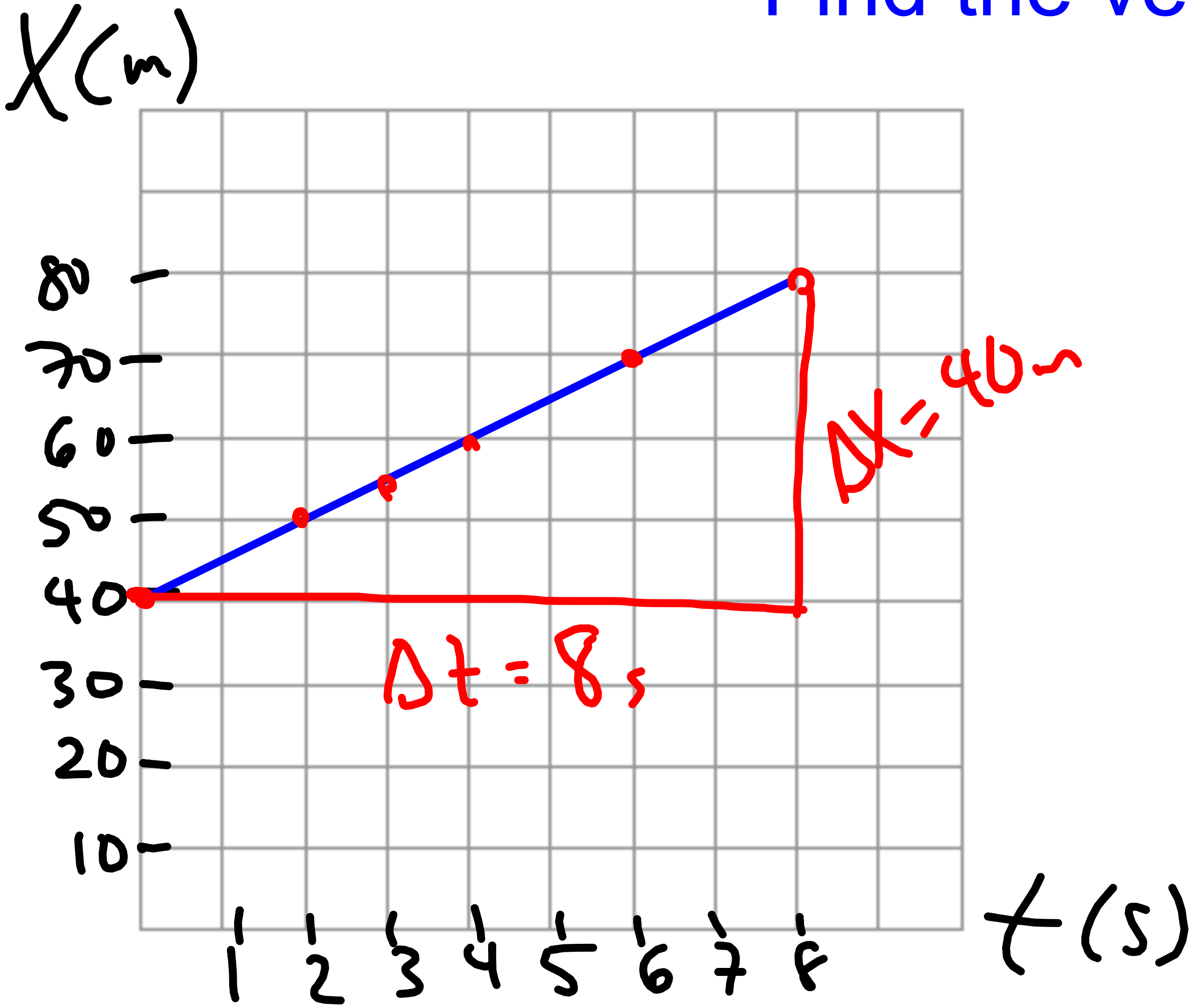
For example, the slope on a position vs time graph is velocity.



$$v_{avg} = \frac{\Delta x}{\Delta t}$$

EX: Finding Slope

Find the velocity at t = 6 s.



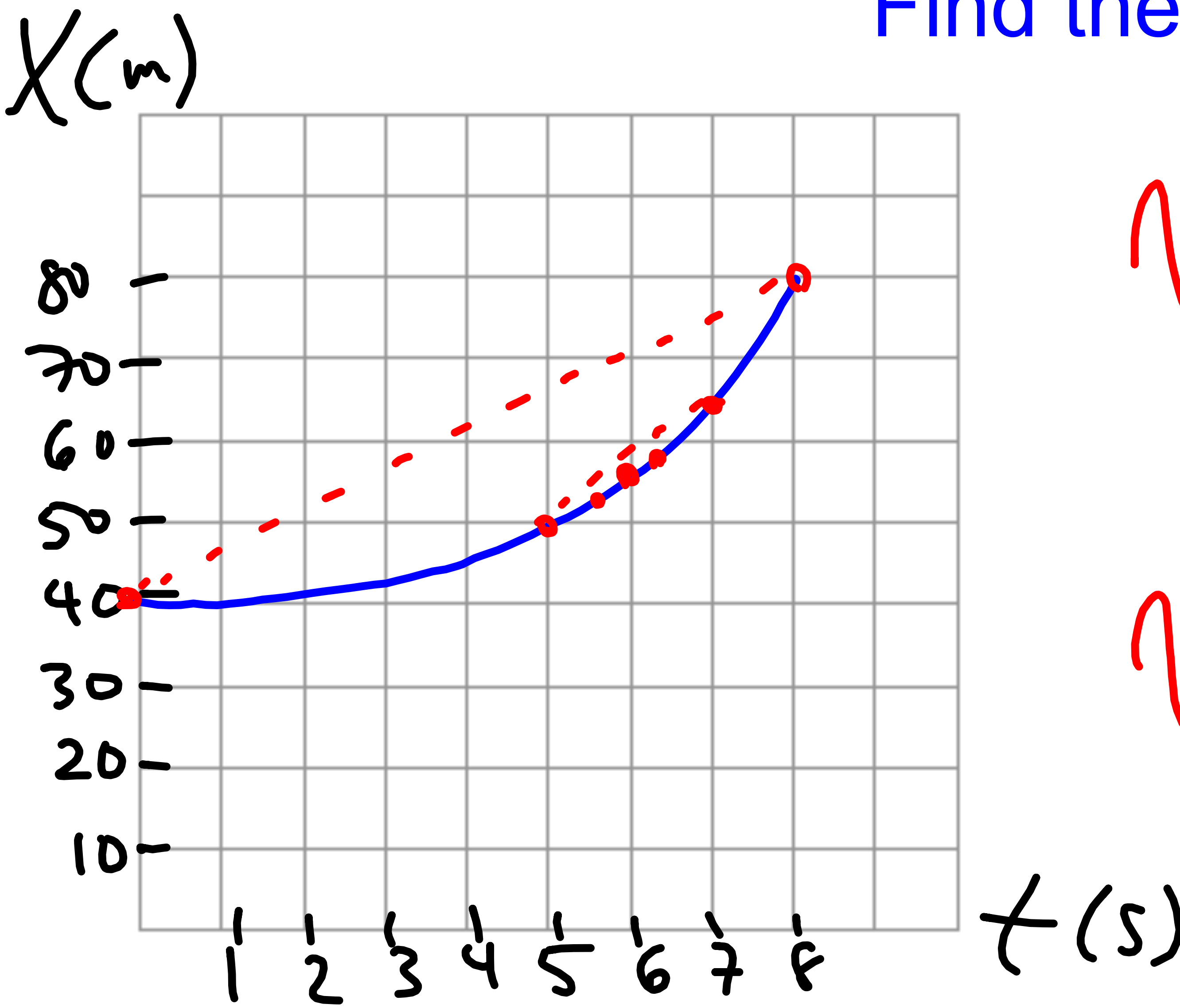
$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$$= \frac{80\,m - 40\,m}{8\,s - 0\,s}$$

$$= \frac{40\,m}{8\,s} = 5\,\frac{m}{s}$$

The Trouble with Non-constant Slopes

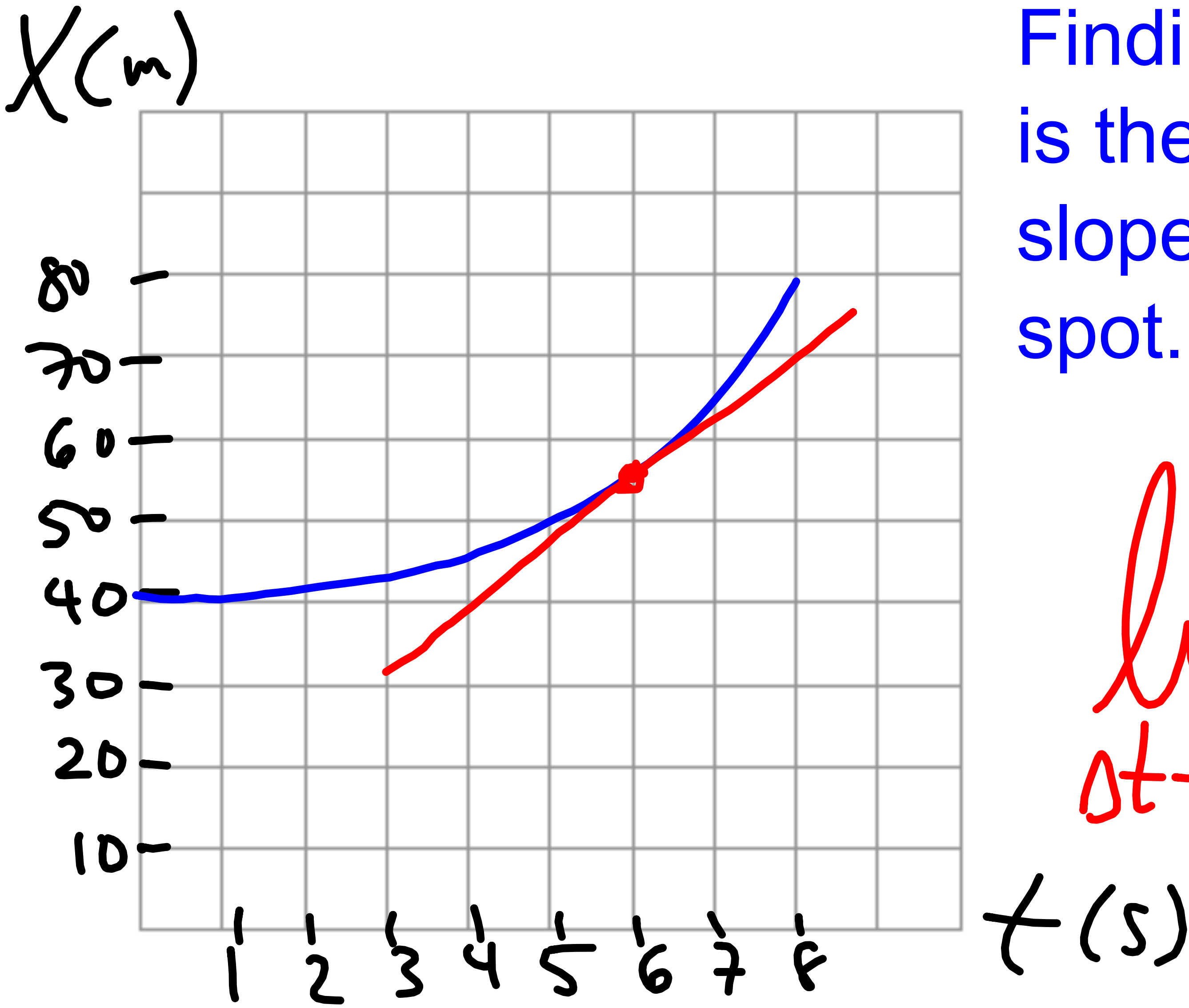
Find the velocity at t = 6 s.



$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$$v = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \right)$$

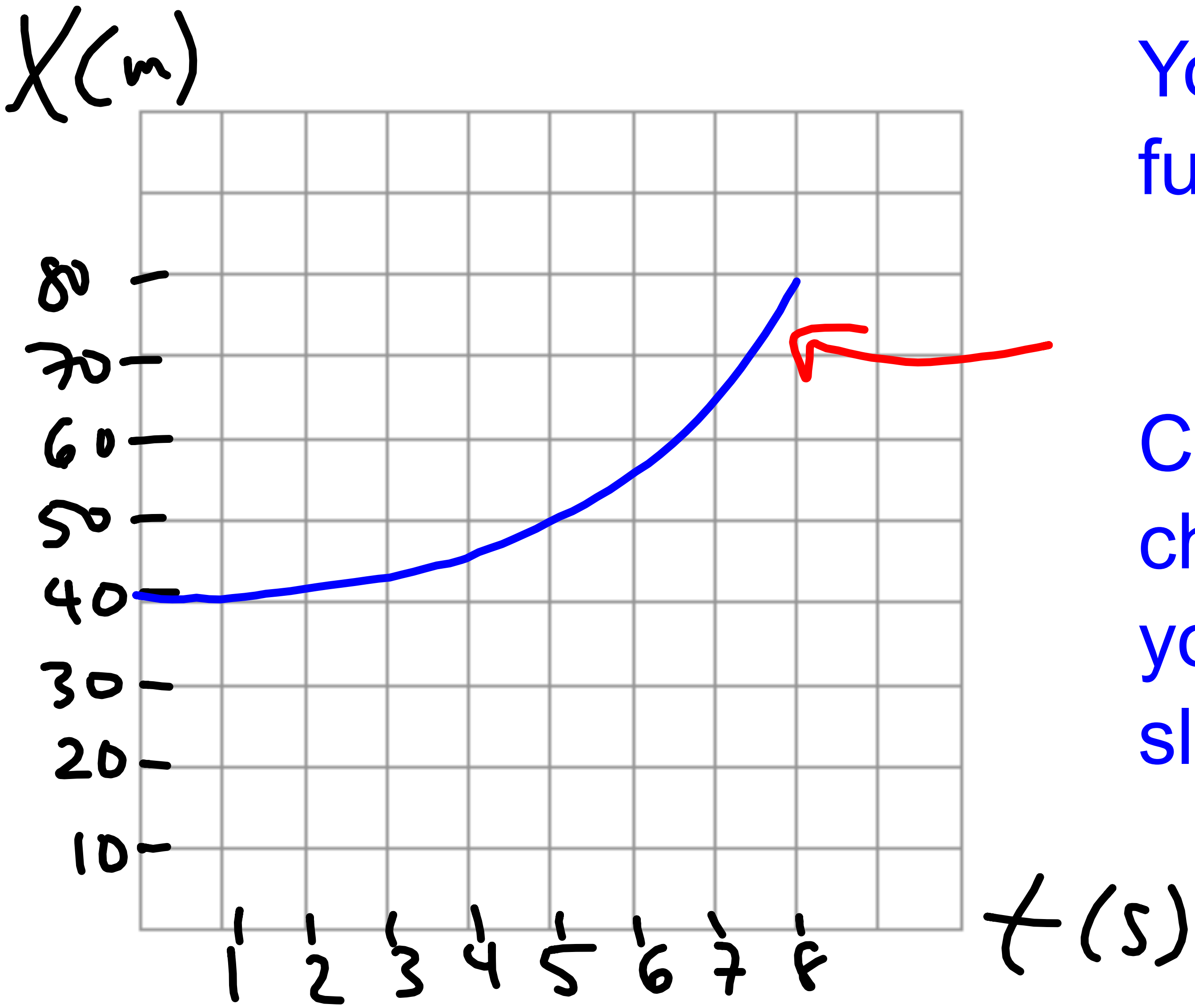
Finding Non-constant Slopes Graphically



Finding the velocity at t = 6 s is the same as finding the slope of a tangent line at that spot.

$$\lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

Finding Non-constant Slopes with Calculus



You need the algebraic function.

Calculus provides a way to change the function so that you get a function for its slope.

Finding Non-constant Slopes with Calculus

"Taking the derivative"

Apply to each term in the function:

- The power comes down and multiplies the coefficient.
- Subtract one from the power.

$$X = 2t^4$$

$$\frac{dx}{dt} = 4 \cdot 2t^{4-1}$$

$$\text{Slope} = \frac{dx}{dt} = 8t^3$$

Finding Non-constant Slopes with Calculus

Special notes:

- Constant terms disappear.
- First power terms lose the variable.

$$x = 3 - 5t^1 + 4t^2 - 2t^3$$

$$\frac{dx}{dt} = 0 - 5 + 8t - 6t^2$$

Physics with Calculus

An object moves according to the equation at right where x is in meters and t is in seconds.

$$x = 4 - 3t + 5t^2 + 2t^3$$

a) What is the initial position of the object?

b) What is its instantaneous velocity at t = 1 s?

c) What is its instantaneous acceleration at t = 1 s?

a) $x(0) = 4 - 3(0) + 5(0)^2 + 2(0)^3$

$= 4\,m$

b) $v = \frac{dx}{dt} = -3 + 10t + 6t^2$

$v(1) = -3 + 10(1) + 6(1)^2 = -3 + 10 + 6$

$= 13\,\frac{m}{s}$

c) $a = \frac{dv}{dt} = 10 + 12t$

$a(1) = 10 + 12(1) = 22\,\frac{m}{s^2}$

$$v = \frac{dx}{dt}$$

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$$a = \frac{dv}{dt}$$

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

Physics with Calculus

An object moves according to the equation at right where x is in meters and t is in seconds.

$$x = 10 + 6t - 2t^2 - 4t^3$$

a) What is the average velocity of the object between t = 0 s and t = 2 s?

b) What is the instantaneous velocity of the object at t = 1 s?

b) $v = \frac{dx}{dt} = 6 - 4t - 12t^2$

$v(1) = 6 - 4(1) - 12(1)^2 = 6 - 4 - 12$

$= -10\,\frac{m}{s}$

a) $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(2) - x(0)}{2 - 0}$

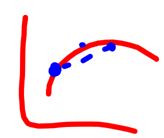
$x(2) = 10 + 6(2) - 2(2)^2 - 4(2)^3$

$x(0) = 10\,m$

$10 + 12 - 8 - 32$

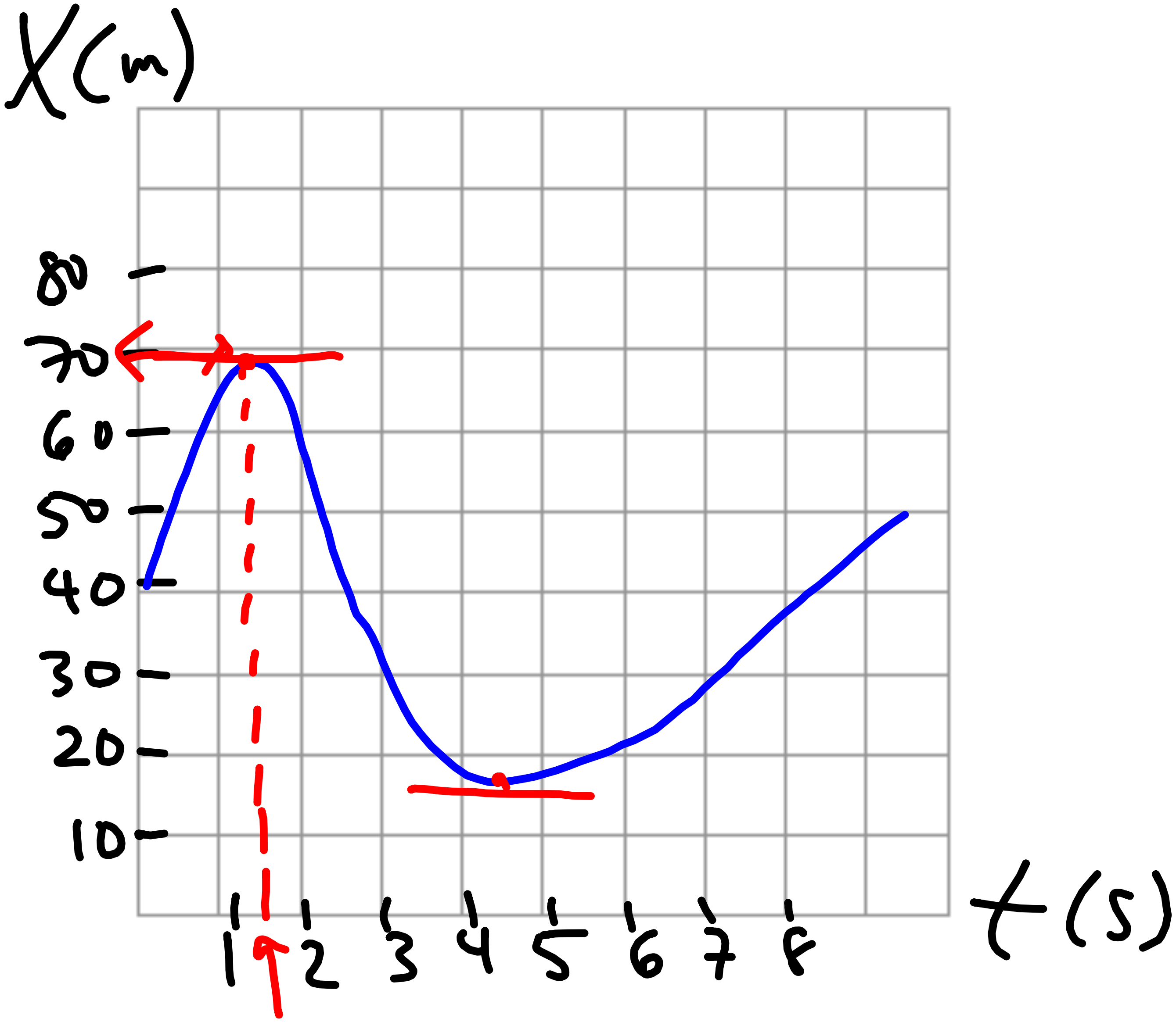
$22 - 40$

$= -18\,m$



$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{-18\,m - 10\,m}{2 - 0} = \frac{-28\,m}{2} = -14\,\frac{m}{s}$

Quick Note About Max and Min Values



At max and min values the slope is zero.

That gives you a way to solve for them with calculus.